PRACTICE PROBLEMS FOR MIDTERM 2

1. Extrema

1.1. Find and classify the critical points of $f(x,y) = x^3y^2(12 - x - y)$.

1.2. Find an classify all the points of the function

$$f(x,y) = (x^2 + y^2)e^{x^2-y^2}$$

1.3. Find the max and min of $f(x,y,z) = x^4 + y^4 + z^4$ on the plane $x + y + z = 1$.

1.4. Find the points on the ellipse

$$23x^2 + 14xy + 23y^2 = 17$$

closest to, and furthest from, the origin: i.e.: maximize the function x^2+y^2 subject to that constraint.

1.5. Find the absolute maxima and minima of the function $f(x,y) = 5x^2-2y^2+10$ on the disk $x^2 + y^2 \leq 1$.

1.6. Find the extrema of $f(x,y) = xy$ subject to the constraints $2x+3y \leq 100, 0 \leq x$, $0 \leq y$.

1.7. Find the extreme points of $f(x,y,z) = x + y + z$ subject to the constraints $x^2 + y^2 = 5$ and $y + 2z = 3$.

1.8. Find the points furthest from and closest to the origin on the curve $x^6+y^6 = 1$.

1.9. The temperature on the spherical surface $x^2 + y^2 + z^2 = 1$ is given by $T(x,y,z) = xy + yz$. Find all the hot spots.

2. Integrals

2.1. **a.** Prove that the volume of the sphere of radius R is

$$2 \cdot \int\int_{D_R} \sqrt{1 - x^2 - y^2}dxdy$$

where D_R is the disk $x^2 + y^2 \leq R^2$.
b. Compute the above integral using polar coordinates.

2.2. Compute $\int\int_D x^2y\,dxdy$ where D is the domain in $\mathbb{R}^2$ bounded by the lines $y = 0$, $y = 1 - x$ and $y = x + 1$.

2.3. Compute

$$\int\int_D xy^2\sqrt{x^2 + y^2}\,dxdy$$

where D is the region of the disk $x^2 + y^2 \leq 1$ where $x \geq 0$ and $y \leq 0$.

2.4. Find the volume under the graph of $f(x,y) = 1 + 2x + 3y$ over the rectangle $[1,2] \times [0,1]$. rectangle.

2.5. $\displaystyle\int\int_{[-1,1]\times[0,1]} (x^2 + y^2)dxdy.$

2.6. Determine the volume of the region in $\mathbb{R}^3$ bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

2.7. Calculate the volume bounded above by the graph $z = 1 - x^3 - y^3$ and below and on the sides by the coordinate planes $x = 0, y = 0, z = 0$.

2.8. Calculate $\int\int\int_T h(x, y, z)dxdydz$, with $h(x, y, z) = x + y + z$ and T the tetrahedron defined by the intersection of the plane $3x + 4y + 5z = 60$ with the first octant (where $x \geq 0, y \geq 0, z \geq 0$).

2.9. Calculate the volume of the solid bounded above by the parabolic cylinder $z = 4 - y^2$ and below by the elliptic paraboloid $z = x^2 + 3y^2$.