Calculus III 110. 202 Midterm 1

Johns Hopkins University

Oct 7, 2009

Name:____

Signature:____

Circle your section number: 1 2 3 4 5 6 7 8 9

Instructions:

1. Not including this cover page, there are 9 pages in this exam. The last two pages are intentionally blank. Feel free to write any of your solutions on the blank pages if necessary, but make sure to give directions to match your solutions and the problems.
2. This is a closed book closed notes exam. No calculators, no collaborations, no talking. Cheating is punished severely.
3. A correct answer but with no argumentations does not guarantee full points. An incorrect answer but with some correct steps always guarantees some partial credits.
4. If you do not understand a question or notation, ask the lecturer or TA.

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Problem 1. [$4 \times 5 = 20$ points] True or false, no argument needed.

(1.1) Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be vectors in $\mathbb{R}^3$ and $\overrightarrow{u} = \lambda \overrightarrow{v}$ for some number λ, then $\overrightarrow{u} \cdot \overrightarrow{v} = |\overrightarrow{u}||\overrightarrow{v}|$.

(1.2) Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be nonzero vectors in $\mathbb{R}^3$. If $\overrightarrow{u} \cdot \overrightarrow{v} = 0$, then $|\overrightarrow{u} \times \overrightarrow{v}| = |\overrightarrow{u}||\overrightarrow{v}|$.

(1.3) A function $f(x, y)$ is differentiable at (x_0, y_0) if both $\frac{\partial f}{\partial x}(x_0, y_0)$ and $\frac{\partial f}{\partial y}(x_0, y_0)$ exist.

(1.4) Let $f(x_1, x_2, \cdots, x_n)$ be a differentiable function whose domain is $\mathbb{R}^n$. If ∇f is the zero vector everywhere, f is a constant function.

Problem 2.[$3 \times 10 = 30$ points]

Let $P = (0, 0, 1), Q = (1, 0, 3)$ and $R = (-1, 2, 1)$ be three points in $\mathbb{R}^3$.

(2.1) Find an equation of the plane which contains the three points P, Q and R.

(2.2) Find the distance from the origin to the plane that you found in (2.1).

(2.3) Find the area of the triangle which has P, Q and R as the three vertices.

3.[20 points] Where is the following function not continuous? Explain your answer.

$$f(x,y) = \begin{cases} \frac{x^2+y^2}{|x|+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

4.[$2 \times 10 = 20$ points]

(4.1) Find the tangent plane of the graph of $z^2 - e^{xy} = 0$ at $(0, 0, 1)$.

(4.2) Find the directional derivative at $(0, 0, 1)$ along the vector $\overrightarrow{v} =< -1, 2, 0 >$ of the function $f(x, y, z) = z^2 - e^{xy}$.

5.[10 points] Find $\frac{df}{dt}(0)$ where $f(x, y, z) = x^3 + y^2 + z$ and $x = \sin t, y = \cos t, z = t^2$.

6.[2 points] **Bonus question.** Directions: 1), do not work on this problem unless you feel confident about all the previous questions; 2), no partial credit and each step in the calculation needs to be explained; 3), your score in this question will be added directly, without curving, to your final score of the course.

Let $f(t)$ be a one variable function which may or may not be continuous. Assume that the right derivative of f at $t = 0$ exists and equals 9, the left derivative of f at $t = 0$ exists and equals -2. Find

$$\lim_{(x,y)\to(0,0)} \frac{f(x^2+y^2) - f(-x^2-y^2)}{x^2+y^2}$$

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