7.4

4. a)

$$A(t) = \iint_D \sqrt{\left(\frac{d(x,y)}{d(u,v)}\right)^2 + \left(\frac{d(y,z)}{d(u,v)}\right)^2 + \left(\frac{d(x,z)}{d(u,v)}\right)^2}\, du\, dv$$

$$\frac{d(x,y)}{d(u,v)} = \det \begin{bmatrix} -(R+\cos\phi)\sin\theta & -\sin\phi\cos\theta \\ (R+\cos\phi)\cos\theta & -\sin\phi\sin\theta \end{bmatrix}$$

$\frac{d(y,z)}{d(u,v)}$ and $\frac{d(x,z)}{d(u,v)}$ same as above!

$$A(T) = \int_0^{2\pi}\int_0^{2\pi} R + \cos\phi \, d\theta\, d\phi = 2\pi\left[R\phi - \sin\phi\right]_0^{2\pi}$$

$$= \boxed{4\pi^2 R}$$

□

b)

$$A(t) = 2\pi \int_{R-1}^{R+1} x\sqrt{1 + \frac{(x-R)^2}{1-(x-R)^2}}\, dx$$

let $u = x - R$

$$= 2\pi\int_{R-1}^{R+1} (u+R)\frac{1}{\sqrt{1-u^2}} = 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin(\theta)+R}{\cos\theta} \times \cos\theta\, d\theta$$

$$= \boxed{4\pi^2 R}$$

6) $z = g(r, y) \quad x = u \quad y = v$

$$A(S) = \iint_D \sqrt{\left(\frac{dg}{dx}\right)^2 + \left(\frac{dg}{dy}\right)^2 + 1}\, dA =$$

$$4 \int_0^2 \int_0^{\sqrt{2-x^2}} \sqrt{y^2 + x^2 + 1}\, dy\, dx = \int_0^{2\pi} \int_0^{\sqrt{2}} \sqrt{r^2+1}\, r\, dr\, d\theta$$

$$= \frac{2\pi}{3}\left[3\sqrt{3} - 1\right]$$

12)

$$\boxed{\begin{aligned} &4\pi r^2 - 2(8\pi - 4\pi\sqrt{3}) \\ &= 8\pi\sqrt{3} \end{aligned}}$$

$$\Phi(t, z) \begin{cases} y = t & -1 \le t \le 1 \\ x = \sqrt{1+t^2} & 0 \le z \le 1 \end{cases}$$

$$A(S) = \iint_D \| T_t \times T_z \|\, dt\, dz = \int_{-1}^{1} \int_0^1 \sqrt{0 + 1 + \left(\frac{t}{\sqrt{1+t^2}}\right)^2}\, dz\, dt$$

24)

a) volume: $4 \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta = \frac{-4\pi}{6}\left(\sqrt{27} - 3\sqrt{3}\right)$

b) $V_{sphere} - V_{cylinder} = \frac{32\pi}{3} - \left(\frac{4\pi}{3}(8 - 3\sqrt{3})\right)$

c)

$$\iint_D \sqrt{1 + f_x^2 + f_y^2}\, dx\, dy = \iint \sqrt{1 + \frac{x^2}{4-x^2-y^2} + \frac{y^2}{4-x^2-y^2}} = \iint \frac{2}{\sqrt{4-x^2-y^2}}$$

$$= \int_0^{2\pi} \int_0^1 \frac{2r}{\sqrt{4-r^2}}\, dr\, d\theta = 8\pi - 4\pi\sqrt{3} \leftarrow$$ surface area of one

7.5

6)

$$4\int_0^2\int_0^{\sqrt{4-x^2}} (x^2+y^2)(4+x+y)\sqrt{3}\,dy\,dx$$

$$=\sqrt{3}\int_0^2\int_0^{2\pi} r^2(4+r\cos\theta+r\sin\theta)r\,d\theta\,dr$$

$$=\boxed{32\pi\sqrt{3}}$$

8) $\vec{N}=2\hat{i}+\hat{j}+\hat{k}$

plane: $2(x-1)+(y-0)+(z-0)=0$

$$\iint_S xyz\,ds=\iint_D \frac{xy(2-2x-y)}{\frac{1}{\sqrt{4+1+1}}}\,dx\,dy=$$

$$\int_0^1\int_0^{-2x+2} xy(2-2x-y)\sqrt{6}\,dy\,dx=$$

$$\sqrt{6}\int_0^1 4x^3-8x^2+4x-8x^4+8x^3-4x^2-\frac{1}{3}(8x^3-16x^2+8x-8x^2+16x-8)\,dx$$

$$=\boxed{\frac{\sqrt{6}}{30}}$$

16)

$$\text{mass} = \iint_S m(x,y,z)\,dS \qquad dS = \frac{dA}{\cos\theta} = \frac{\|\vec{N}\|}{N\cdot k}\,dA$$

$$\|\vec{N}\| = \left( \frac{x^2}{R^2-x^2-y^2} + \frac{y^2}{R^2-x^2-y^2} + 1 \right)^{\frac{1}{2}}$$

$$\vec{\ }\text{mass} = \iint_D (x^2+y^2)\left(\frac{R^2}{R^2-x^2-y^2}\right)^{\frac{1}{2}} dx\,dy =$$

$$\int_0^{2\pi}\int_0^R \underbrace{R\,\frac{r^3}{\sqrt{R^2-r^2}}}_{\text{basic Trigeometric substitution}}\,dr\,d\theta = \boxed{\frac{4}{3}\pi R^4}$$

23) a)

$$E = \left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2$$

$$G = \left(\frac{dx}{dv}\right)^2 + \left(\frac{dy}{dv}\right)^2 + \left(\frac{dz}{dv}\right)^2$$

$$F = \frac{d^2x}{du\,dv} + \frac{d^2y}{du\,dv} + \frac{d^2z}{du\,dv}$$

$$\rightarrow \|T_u \times T_v\| = \sqrt{EG-F^2}$$

$$\|T_u \times T_v\| = \det \begin{vmatrix} i & j & k \\ \frac{dx}{du} & \frac{dy}{du} & \frac{dz}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} & \frac{dz}{dv} \end{vmatrix}$$

b)

$$A(S) = \iint_D \sqrt{EG}\, du\, dv$$

c)

$$A(S) = \iint_D \left\| \frac{\partial \Phi}{du} \right\| \left\| \frac{d\Phi}{dv} \right\| du\, dv = \underline{\underline{\iint_D a^2 \sin\phi \, d\phi \, d\theta}}$$

7.6

2)

$$\iint_D F(\Phi(u,v)) \cdot (T_u \times T_v)\, du\, dv =$$

$$\int_0^{2\pi}\int_0^1 (2\sin u,\ 3\cos u,\ v^2) \cdot \det \begin{vmatrix} i & j & k \\ 2\cos u & -3\sin u & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \int_0^{2\pi}\int_0^1 -6\sin^2 u - 6\cos^2 u \, dv\, du = \boxed{-\frac{12}{\pi}}$$

6) ~~Sto~~ Use standard parametrization of sphere

$$\int_0^{\pi}\int_0^{2\pi} -\nabla T\, ds = \int_0^{\pi}\int_0^{2\pi} \cos\theta \times \sin\phi + \sin\phi \sin\theta + \cos\phi \, d\theta\, d\phi$$

$$= \underline{\underline{0}}$$

$\square$

26)

In: $$\iint_S \frac{1}{\|\vec{x}-\vec{p}\|} ds = \int_0^{\pi}\int_0^{2\pi} \frac{1}{\|x-p\|} r^2 \sin\phi \, d\theta \, d\phi$$

$$= \frac{2\pi r}{\|x-p\|} (-\cos\phi)\Big|_0^{\pi} = 4\pi r$$

Out:

$$\frac{4\cdot \pi r^2}{d}$$

14)

~~[illegible]~~

normal vector to barrel $(x, y, 0)$

" " " to $(0, 0, 1)$

barrel:
$$\iint_S (1, 1, z(x^2+y^2)^2) \cdot (x, y, 0)\, dx dy$$

$$= \int_0^{2\pi}\int_0^1 r^2 \cos\theta + r^2 \sin\theta \, dr d\theta = 0$$

Top: $\iint_S z(x^2+y^2)\, ds \quad z = 1$

$$= \int_0^{2\pi}\int_0^1 r^3 \, dr d\theta = \boxed{\frac{\pi}{3}}$$