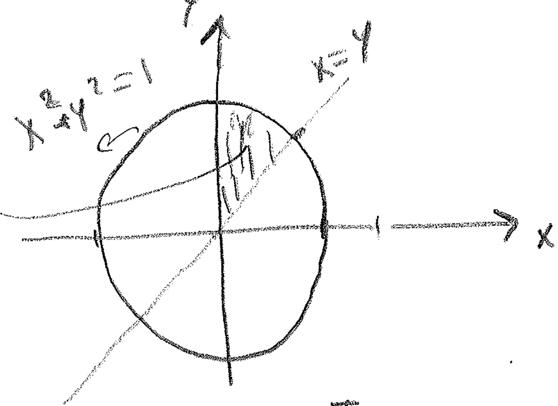
6.2.28)

$$\iint_D x^2\,dx\,dy$$

$$D = \begin{cases} 0 \le r \le 1 \\ \pi/4 \le \theta \le \pi/2 \end{cases}$$



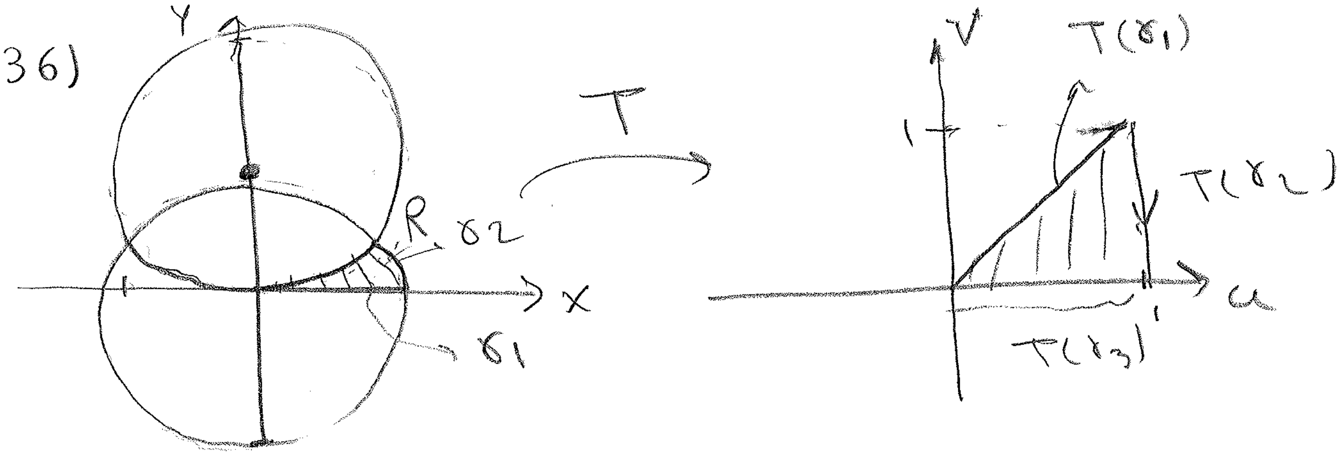
$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases} \Rightarrow dx\,dy = r\,dr\,d\theta \Rightarrow \iint_D x^2\,dx\,dy = \int_0^1 \int_{\pi/4}^{\pi/2} r^2\cos^2(\theta)\, r\,dr\,d\theta$$

$$= \frac{1}{4}\int_{\pi/4}^{\pi/2} \frac{1+\cos(2\theta)}{2}\,d\theta = \frac{\theta}{8}\Big|_{\pi/4}^{\pi/2} + \frac{\sin(2\theta)}{4\times 4}\Big|_{\pi/4}^{\pi/2}$$

$$= \boxed{\frac{\pi}{32} - \frac{1}{16}}$$

6.2.36)

a)



b) To identify Domain D in uv plane we look at the Image of boundaries of R:

$\gamma_1 = \{(x,0)\} \xrightarrow{T} \{(x^2, x^2)\} = \{(u,v) : u = v\} = T(\gamma_1)$

$\gamma_2 = \{(x,y) : x^2+y^2 = 1\} \xrightarrow{T} \{(1, 1-2y)\} = \{(u,v) : u = 1, 0 \le v \le 1\}$

$\gamma_3 = \{(x,y) : x^2+y^2 = 2y\} \xrightarrow{T} \{(2y, 0)\} = \{(u, 0) : 0 \le u \le 1\}$

c) $dx\,dy = \dfrac{\partial(x,y)}{\partial(u,v)}\,du\,dv = \dfrac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}}\,du\,dv = \dfrac{1}{\begin{pmatrix} 2x & 2y \\ 2x & 2y-2 \end{pmatrix}}\,du\,dv$

$= \dfrac{1}{(4xy - 4x - 4xy)}\,du\,dv = \dfrac{-1}{4x}\,du\,dv$

and $y = \dfrac{u-v}{2}$ so

$\iint_R x e^y\,dx\,dy = \iint_D x e^y \dfrac{-1}{4x}\,du\,dv = -\iint_D e^{\frac{u-v}{2}} \dfrac{du\,dv}{4} = \int_0^1 \int_0^u e^{\frac{u-v}{2}}\,dv\,du$

$\rightarrow \int^1 \left(-2e^{\frac{u-v}{2}}\Big|^u\right) du = -2\int^1 \left(1 - e^{u/2}\right) du = \boxed{-2(1 - 2e^{1/2} + 2)}$

7.1.8)

$$\begin{cases} y^2+z^2=1 \\ z=x \end{cases}, \text{ Let } \quad C(t): [0,1] \longrightarrow \mathbb{R}^3$$

$$C(t) = (t, \pm\sqrt{1-t^2}, t)$$

7.1.12)

a) $C(t) = (t, t^2, 0) \Rightarrow \begin{cases} C'(t) = (1, 2t, 0) \Rightarrow \|C'(t)\| = \sqrt{1+4t^2} \\ f(C(t)) = f(t, t^2, 0) = t \end{cases}$

$$\Rightarrow \int_C f\,ds = \int_0^1 t\sqrt{\underbrace{1+4t^2}_{u}}\,dt = \frac{1}{8}\,\frac{(1+4t^2)^{3/2}}{3/2}\Big|_0^1 = \frac{1}{12}(1+4t^2)^{3/2}\Big|_0^1$$

$$= \boxed{\frac{1}{12}\left(5\sqrt{5}-1\right)}$$

---

b) $C(t) = (t, \frac{2}{3}t^{3/2}, t) \Rightarrow \begin{cases} C'(t) = (1, \sqrt{t}, 1) \Rightarrow \|C'(t)\| = \sqrt{2+t} \\ f(C(t)) = \dfrac{t+\frac{2}{3}t^{3/2}}{\frac{2}{3}t^{3/2}+t} = 1 \end{cases}$

$$\int_C f\,ds = \int_1^2 \sqrt{2+t}\,dt = \frac{2}{3}(t+2)^{3/2}\Big|_1^2 = \boxed{\frac{2}{3}\left(8-3\sqrt{3}\right)}$$

7.1.18)

$c(\theta) = (0, a\sin\theta, a\cos(\theta))$, $0 \le \theta \le \pi$

$\delta(x,y,z) = 2$

$$\Rightarrow \int_C \delta\, ds = 2\int_0^{\pi} \|c'(\theta)\|\, d\theta = \boxed{2\pi a} \rightarrow \text{Total mass}$$

$$\bar{y} = \frac{\int_C y\delta\, ds}{\int_C \delta\, ds} = \frac{2a^2\int_0^{\pi} \sin(\theta)\, d\theta}{2\pi a} = \frac{2a^2}{2\pi a}\left(-\cos(\theta)\Big|_0^{\pi}\right) = \boxed{\frac{2a}{\pi}}$$

7.2.4

a) $$\int_C x\,dy - y\,dx = \int_0^{2\pi} (\cos^2(t) + \sin^2(t))\,dt = \boxed{2\pi}$$

b) $$\int_C x\,dx + y\,dy = \int_0^{2} +\pi\left(-\sin(\pi t)\cos(\pi t) + \sin(\pi t)\cos(\pi t)\right)dt = 0$$

d) $c(t) = (t, 0, t^2)$, $-1 \le t \le 1$

$$\Rightarrow \int_C x^2\,dx - xy\,dy + dz = \int_{-1}^{1} (t^2 + 2t)\,dt = \boxed{\frac{2}{3}}$$

7.2.6

a) F is perp to $c'(t) \iff \overbrace{F(c_1(t), c_2(t), c_3(t))}^{c(t)} \cdot \overbrace{(c_1'(t), c_2'(t), c_3'(t))}^{c'(t)} = 0$

for every t

so this implies

$$\int_C F \cdot ds = \int_a^q \underbrace{F(c_1, c_2, c_3)}_{F(c(t))} \cdot \underbrace{(c_1', c_2', c_3')}_{c'(t)} \, dt = 0$$

b) if F is parallel to c then $F(c(t)) \cdot c'(t) = \|F(c(t))\| \|c'(t)\|$

so it means

$$\int_C F \cdot ds = \int_0^1 F(c(t)) \cdot c'(t) dt = \int_a^1 \|F\| \underbrace{\|c'(t)\| dt}_{ds}$$

$$= \int_S \|F\| \, ds$$

7.2.8) $\begin{cases} F(x,y,z) = (y, 2x, y) \\ c(t) = (t, t^2, t^3) \end{cases} \Rightarrow \begin{cases} F(c(t)) = (t^2, 2t, t^2) \\ c'(t) = (1, 2t, 3t^2) \end{cases}$

$\Rightarrow F \cdot ds = (t^2, 2t, t^2) \cdot (1, 2t, 3t^2) = t^2 + 4t^2 + 3t^4 = 3t^4 + 5t^2$

$\rightarrow \int_C F \cdot ds = \int_a^1 (3t^4 + 5t^2) \, dt = \boxed{\frac{3}{5} + \frac{5}{3}}$

7.2.18)

$\nabla f = (2xyze^{x^2}, ze^{x^2}, ye^{x^2})$

Let $C(t) = (t, t, 2t)$, $C(0) = (0,0,0)$, $C(1) = (1,1,2)$

$$f(1,1,2) - f(0,0,0) = \int_C \nabla f \cdot ds = \int_0^1 e^{t^2}(4t^3 + 2t + 2t)\,dt$$

$$= 4\int_0^1 t^3 e^{t^2} dt + 4\int_0^1 e^{t^2} t\,dt = 2t^2 e^{t^2}\Big|_0^1 - \overbrace{4\int_0^1 t e^{t^2} + 4\int_0^1 e^{t^2} t\,dt}$$

$$= 2e$$

$$\rightarrow f(1,1,2) = f(0,0,0) + 2e = \boxed{5 + 2e}$$

7.2.20

Let $C(t) = (c_1(t), c_2(t), c_3(t))$ be the path that cyclist rides along. Then $c_3'(t) = \text{Constant} = 2\pi$

$$\int_C F \cdot ds = \int_0^1 (c_2, c_1, 1) \cdot (c_1', c_2', k)\,dt = \int_0^1 ((c_1 c_2)' + 2\pi)\,dt$$

$$= (c_1 c_2)(t)\Big|_0^1 + 2\pi t\Big|_0^1 = (c_1(1)c_2(1) - c_1(0)c_2(0)) + 2\pi = 0 + 2\pi = \boxed{2\pi}$$

7.3.8)

(a) $\longrightarrow$ (i)

(b) $\longleftrightarrow$ (iii) $\longrightarrow$ $Z = 4 + X^2 + Y^2$

(c) $\leftrightarrow$ (ii) $\longrightarrow$

(d) $\longleftrightarrow$ (iv) $\rightsquigarrow$ $X^2 + Z^2 = \left((y+3)^2 + 2\right)^2$

7.3.12

$$[-\pi, \pi] \times [-\pi, \pi] \longrightarrow \left((2-\cos(v))\cos(u),\ (2-\cos(v))\sin(u),\ \sin(v)\right)$$

$$T_u = \frac{\partial \phi}{\partial u} = \left(-2\sin(u) + \cos(v)\sin(u),\ 2\cos(u) - \cos(v)\cos(u),\ 0\right)$$

$$T_v = \frac{\partial \phi}{\partial v} = \left(\sin(v)\cos(u),\ \sin(v)\sin(u),\ \cos(v)\right)$$

$$\Rightarrow T_u = (\cos(v) - 2)\left(\sin(u),\ \cos(u),\ 0\right)$$

$$\rightarrow T_u \times T_v = (\cos(v) - 2)\left(-\cos(v)\cos(u),\ -\cos(v)\sin(u),\ \sin(v)\right)$$

$$\boxed{\vec{n} = \frac{T_u \times T_v}{\|T_u \times T_v\|} = \left(\cos(v)\cos(u),\ \cos(v)\sin u,\ \sin(v)\right)}$$

it is a regular surface