MATH 202: Homework 8

due Wednesday, November 8

A subset D of $\mathbf{R}^2$ is an *x-simple region* if we can describe it as

$$D = \{(x, y) \in \mathbf{R}^2 \mid a \leq x \leq b \text{ and } \varphi_1(x) \leq y \leq \varphi_2(x)\}$$

for some interval $[a, b]$ and continuous mappings $\varphi_1, \varphi_2 \colon [a, b] \longrightarrow \mathbf{R}$. Similarly, we say that D is a *y-simple region* if we can describe it as

$$D = \{(x, y) \in \mathbf{R}^2 \mid \psi_1(y) \leq x \leq \psi_2(y) \text{ and } c \leq y \leq d\}$$

for some interval $[c, d]$ and continuous mappings $\psi_1, \psi_2 \colon [c, d] \longrightarrow \mathbf{R}$.

(1) For each of the following, explicitly describe D as an x-simple region and as a y-simple region. If D itself is not x-simple or y-simple, split it into subsets that are x-simple or y-simple and describe those subsets.

   (a) D is the triangle in $\mathbf{R}^2$ with vertices $(-1, -1)$, $(1, -1)$ and $(0, 2)$

   (b) D is the portion of the unit disk centered at the origin in $\mathbf{R}^2$ with $y \geq 0$

   (c) $D = \{(x, y) \in \mathbf{R}^2 \mid 2 \leq |(x, y)| \leq 3\}$

   (d) $D = ([0, 1] \times [0, 1]) \smallsetminus ((1/3, 2/3) \times (1/3, 2/3))$

   (e) D is the region in between the curves $x - y = 0$ and $x = y^2 - 2$

(2) For each region D in (1), use Fubini's theorem to write down $\int_D f$ as an iterated integral using both possible orders of integration.

Do the following problems from the textbook:

§6.5: 4, 5, 9(a)(c)(e)

§6.6: 1, 3, 4, 5(a)(c), 6, 7, 8