MATH 202: Homework 3

due Wednesday, September 20

Skim through §3.1–3.6 of the text. Based on your knowledge from Math 201, this should be fairly easy, but the later sections may include some material that is new to you.

(1) Which of the following sets have the structure of a vector space? You need not justify your answer.

   (a) The set $\mathcal{L}(\mathbf{R}^n, \mathbf{R}^m)$ of all linear mappings $L\colon \mathbf{R}^n \longrightarrow \mathbf{R}^m$.

   (b) The set $\mathcal{A}(\mathbf{R}^n, \mathbf{R}^m)$ of all affine mappings; an affine mapping is a mapping of the form $L(x) + p$, for some $L \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^m)$ and $p \in \mathbf{R}^m$.

   (c) The set $\mathrm{M}_{m,n}(\mathbf{R})$ of $m \times n$ matrices.

   (d) The set of invertible linear mappings $L\colon \mathbf{R}^n \longrightarrow \mathbf{R}^m$.

(2) What is the dimension of each vector space that you identified in (1)?

(3) Fill in the blank: *A square matrix is invertible if and only if its echelon form is* _____.

(4) Let

$$M = \begin{bmatrix} 1 & 2 & 5 & 2 \\ -1 & 0 & 8 & -4 \\ 2 & 4 & 1 & 7 \end{bmatrix}$$

   Compute $R_{2;3,-1}M$, $S_{1,2}M$ and $T_{2;3}M$.

(5) Use the universal characterization of the determinant to prove that $\det(AB) = \det(A)\det(B)$.

Then do the following problems from the textbook, using additional reading and the lectures as preparation.

§3.5: 6

§3.6: 2

§3.8: 3, 4, 7

§3.9: 2, 3

§3.10: 4