MATH 202: Homework 12

due Friday, December 8

(1) Let γ be a parametrization of the boundary of the square with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$, oriented in the clockwise direction. (a) Evaluate $\int_\gamma (x^2-y^2)dx+(x^2+y^2)dy$ by computing it as a flow integral. (b) Evaluate the same integral using Green's theorem.

(2) Let S be the surface defined by $x^2+y^2=z^2$ for $1 \le z \le 3$ and $x^2+y^2=1$ for $0 \le z \le 1$, and let $F(x,y,z)=(2yz,-2xz,1)$. (a) Find a vector field G for which $\nabla \times G = F$. (b) Equip S with the outwards pointing orientation $\widehat{N}$ and compute

$$\int_S F \cdot \widehat{N}$$

using Stokes' theorem. Compare with your answer to problem (4) of HW#11.

(3) Let

$$F(x,y,z)=\frac{x\mathbf{e}_1+y\mathbf{e}_2+z\mathbf{e}_3}{\sqrt{x^2+y^2+z^2}}$$

and

$$W=\{(x,y,z)\in \mathbf{R}^3 \mid a^2 \le x^2+y^2+z^2 \le b^2\}.$$

Evaluate $\int_W \nabla \cdot F$ using Gauss's theorem.

(4) Use Stokes's theorem to find the work done by the vector field

$$F(x,y,z)=(xyz-e^x,-xyz,x^2yz+\sin z)$$

on a particle that moves along the line segments from $(0,0,0)$ to $(1,1,1)$, then to $(0,0,2)$, and finally back to $(0,0,0)$.

(5) Let D be the unit disk in $\mathbf{R}^2$ centered at the origin, and let Φ be the 2-surface $\Phi\colon D \longrightarrow \mathbf{R}^3$ given by

$$\Phi(x,y)=(x,y,(1-x^2-y^2)e^{1-x^2-3y^2}).$$

Consider the 2-form

$$\omega=(x^2+y^2+3)\,dx\wedge dy+\sqrt{x^3+1}\sin z\,dz\wedge dx+e^y\cos z\,dy\wedge dz.$$

Evaluate $\int_\Phi \omega$. (Warning: do not attempt to compute this directly. Use one of the integral theorems.)

Do the following problems from the textbook:

§9.9: 2, 3(a)(b)(c), 4

§9.16: 2, 4