MATH 202: Homework 11

due Wednesday, November 29

(1) Find the volume of the solid W bounded by the surfaces $z = 4x^2 + y^2$ and $y^2 + z = 2$.

(2) Calculate the flux of the vector field $F(x, y, z) = (x, y, z)$ through the sphere S of radius a, oriented with the inwards pointing unit normal vector.

(3) Consider the parametrized 2-surfaces in $\mathbf{R}^3$

$$\Phi\colon [0, 2] \times [0, 2\pi] \longrightarrow \mathbf{R}^3$$
$$\Phi(u, v) = (u \cos v, u \sin v, 3u^2)$$

and

$$\Psi\colon [0, 1] \times [0, 4\pi] \longrightarrow \mathbf{R}^3$$
$$\Psi(u, v) = (2u \cos v, 2u \sin v, 12u^2)$$

(a) Argue that the image of Φ and the image of Ψ are the same surface in $\mathbf{R}^3$ (Hint: give equations in (x, y, z) for the surfaces parametrized by Φ and Ψ.)

(b) Evaluate

$$\int_\Phi F \cdot \widehat{N} \qquad \text{and} \qquad \int_\Psi F \cdot \widehat{N}$$

where $F(x, y, z) = (y, -x, z^2)$. Why are your answers different?

(4) Let S be the surface defined by $x^2+y^2 = z^2$ for $1 \leq z \leq 3$ and $x^2+y^2 = 1$ for $0 \leq z \leq 1$. (a) Sketch S. (b) Find the flux of the vector field $F(x, y, z) = (2yz, -2xz, 1)$ through the surface S, where S is equipped with the outwards pointing orientation. (Hint: first write down the unit normal vector explicitly on each piece of S.)

Do the following problems from the textbook:

§9.5: 1, 2

§9.7: 2

§9.8: 1, 2, 3, 4, 5