MATH 202: Homework 1

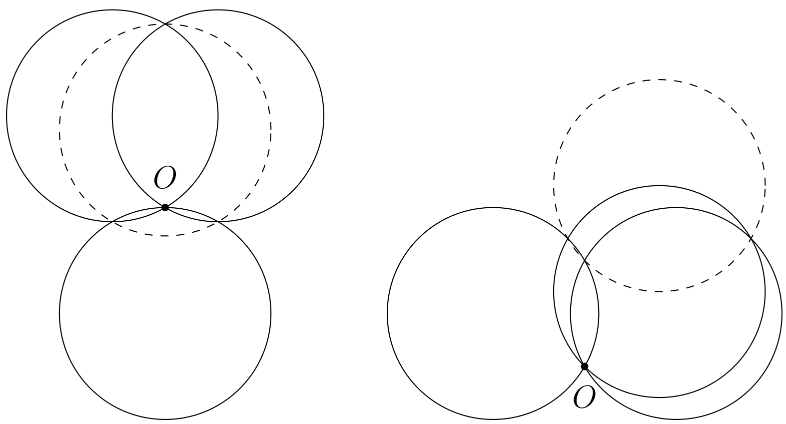
due Wednesday, September 6

Do the following problems from the textbook:

§2.2: 2, 3, 8, 10

§2.3: 1, 2, 4, 8, 9

For the next problems, suppose that three circles of equal radius r intersect at a common point O. Here are two examples.

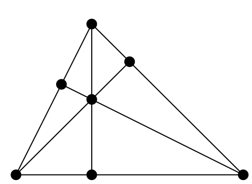


Let X_1, X_2, and X_3 denote the centers of the circles, and for $i = 1, 2, 3$ let $\mathbf{x}_i$ be the vector $\overrightarrow{OX_i}$. Let A, B, and C denote the three intersection points besides O; explicitly, A is the other intersection point of the circles centered at X_2 and X_3, B is the other intersection point of the circles centered at X_1 and X_3, and C is the other intersection point of the circles centered at X_1 and X_2. Define $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$.

(i) Express the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in terms of the vectors $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$.

(ii) Show that A, B and C lie on a circle of the same radius r as the given circles. In the examples above this is the dashed circle. (Hint: what is its center?)

(iii) An *altitude* of a triangle is a line through one of the vertices that is perpendicular to the line through the other two vertices. The *orthocenter* of a triangle is the common intersection point of the three altitudes:



Show that O is the orthocenter of the triangle ABC.