MATH 112 MIDTERM 2 PRACTICE QUESTIONS

1. Prove that for any ordered field, $0 < 1$. (Here 0 and 1 are additive an multiplicative identities respectively).

2. For each of the following subsets of $\mathbb{R}$ determine min, max, sup, inf, if applicable. Justify your answer.
   (a) $\{\sqrt{2}, 1, e, \pi\}$
   (b) $\{x \in \mathbb{R} \colon 2 \leq x^2 < 5\}$
   (c) $\{x \in \mathbb{Q} \colon 2 \leq x^2 < 5\}$
   (d) $\{2n/n + 2 \colon n \in \mathbb{N}\}$

3. Let S and T be subsets of a complete ordered field F. Define $S + T := \{s + t \colon s \in S,\ t \in T\}$. Prove that if S and T are bounded above, then $\sup(S + T) \leq \sup(S) + \sup(T)$.

4. Let A and B be two open subsets of F ($F = \mathbb{R}$ or $\mathbb{C}$). Prove the following
   (a) $A \cup B$ is open
   (b) $(A \cup B)^c = A^c \cap B^c$
   (c) $A^c \cap B^c$ is closed

5. Determine if the following sets are open, closed, or neither.
   (a) $\{x \in \mathbb{R} \colon x^2 < 2\} \subseteq \mathbb{R}$
   (b) $\{x \in \mathbb{Q} \colon x^2 < 2\} \subseteq \mathbb{R}$
   (c) $\{x \in \mathbb{C} \colon |x - (1 + i)| < 5\} \subseteq \mathbb{C}$
   (d) $\{x \in \mathbb{C} \colon |Re(x) - Re(1 + i)| < 5\} \subseteq \mathbb{C}$
   (e) $\bigcup_{k=1}^{\infty}(1, 1 + 1/k) \subseteq \mathbb{R}$

6. Write the following numbers in the polar form
   (a) i^3
   (b) $1 + i$
   (c) $(1 + i)^7$
   (d) $5 - 4i$

7. Prove that for any $z \in \mathbb{C}$, $z/|z|$ is a unit vector.

8. Let $\{s_n\}$ be a sequence of complex numbers, such that for each $n \in \mathbb{N}$ s_n lies on the unit circle. Prove that s_n does not converge to 0.

9. Carefully justifying every step, calculate the following limits (or prove that the sequence diverges)
   (a) $\lim 1/\sqrt{n}$
   (b) $\lim \frac{-3n^2+2}{n^3-4n+1}$
   (c) $\lim n^n$
   (d) $\lim \frac{n}{n!}$
   (e) $\lim((n + 1)^2 - n^2)$