

MATH 112 MIDTERM 1 PRACTICE QUESTIONS

1. Prove that for all $n \geq 0$

$$\sum_{k=0}^n 2^k(k+1) = 2^{n+1}n + 1.$$

2. Let $A, B, C \subseteq U$ be sets. Prove the following

- (a) $(A \cap C) \setminus B = (A \setminus B) \cap (C \setminus B)$.
(b) $U \setminus (A \setminus B) = (U \setminus A) \cup B$.

3. Prove

$$\bigcup_{k \in \mathbb{N}} [-1/k, 0] = [-1, 0].$$

4. Let R be a relation on $\mathbb{R} \times \mathbb{R}$ given by $(a, b)R(c, d)$ if $a - c$ and $b - d$ are integers.

- (a) Prove that R is an equivalence relation.
(b) Prove that for any $(a, b) \in \mathbb{R} \times \mathbb{R}$ there exists $(c, d) \in [0, 1) \times [0, 1)$ such that $[(a, b)] = [(c, d)]$.

5. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x + 2$, and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = x^3$. Determine the following

- (a) What is the range of f ? Of g ?
(b) Is f surjective, injective, bijective? Same questions for g .
(c) If f or g is bijective, find the inverse.
(d) Find $g \circ f$ and $f \circ g$.
(e) What is the range of $g \circ f$? Of $f \circ g$?
(f) Is $g \circ f$ surjective, injective, bijective? Same questions for $f \circ g$.

6. Let F be an ordered field, $x, y \in F$. Prove the following

- (a) $-x - y = -(x + y)$.
(b) $(-1)x = -x$.
(c) $xy = 0$ iff $x = 0$ or $y = 0$.
(d) If 0_1 and 0_2 are two additive identities, then $0_1 = 0_2$.

7. Compute the following in \mathbb{Z}_5

- (a) $\bar{7}^3 + \bar{38} - \bar{25} \cdot \bar{349}$
(b) $\bar{8}^{10}$

(Here, $\mathbb{Z}_5 = \mathbb{Z}/5\mathbb{Z}$ and $\bar{a} = [a]$, the equivalence class modulo 5. Your solutions should be represented by integers between 0 and 4.)