Math 112 Homework for Tuesday, Week 13

1. Last Friday, we proved that a complex power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ satisfies one of the following:

   (a) $f(z)$ converges only when $z = 0$.

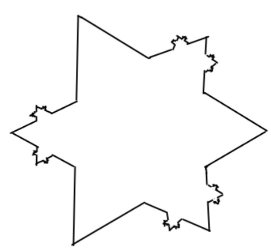
   (b) $f(z)$ converges for all $z \in \mathbb{C}$.

   (c) There exists a positive real number R such that $f(z)$ converges absolutely for $|z| < R$ and diverges for $|z| > R$.

   Carefully review the proof of this theorem given in class, then explain why it is the case that if $f(z)$ converges for all $z \in \mathbb{C}$, then it converges absolutely for all $z \in \mathbb{C}$. (The result follows from something used on Friday to prove the main theorem stated above. Start like this: "Suppose that $f(z)$ converges for all $z \in \mathbb{C}$. Let $z \in \mathbb{C}$." Then explain why $f(z)$ converges absolutely.)

2. Use the theorem about differentiation of power series to prove that $\sin'(z) = \cos(z)$ and $\cos'(z) = -\sin(z)$ for all $z \in \mathbb{C}$.

3. Start with an equilateral triangle with edge-length 1. In the middle of each edge of the triangle, add a triangle of edge-length $1/3$. This gives three new triangles. In the middle of each of the two outer edges of these triangles, add an equilateral triangle of edge-length $1/3^2$. In total, we've just added 6 of these new smaller triangles. In the middle of each of the outer edges of these, add an equilateral triangle of side-length $1/3^3$. Repeat, forever. The resulting figure looks vaguely like this:

   What is the perimeter of this figure?