

Math 112 Homework for Friday, Week 11

1. Using the tests from the handout, state whether each of the following series converges or diverges. Justify your solution with the name of the test and any relevant calculation.

(a) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

(c) $\sum_{n=1}^{\infty} \frac{n}{n^3 - 5}$

(d) $\sum_{n=1}^{\infty} \frac{n!}{(2^n)^3}$

(e) $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$

(f) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

Warning: the series in (f) is not a p -series.

2. State, with justification, whether each of the following series converges absolutely, converges conditionally, or diverges.

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(b) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

(d) $\sum_{n=0}^{\infty} \frac{(-2)^n}{5^{n+1}}$

(e) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

3. Let $\{a_n\}$ be a complex sequence, and $c \in \mathbb{C}$, $c \neq 0$. Is it true that $\sum_{k=1}^{\infty} a_k$ converges if and only if $\sum_{k=1}^{\infty} ca_k$ converges? If true, prove; if false, give a counterexample.

4. Find the following limits and provide an ε - δ proof:

(a) $\lim_{x \rightarrow 1} \frac{1}{x+1}$

(b) $\lim_{x \rightarrow 5} \sqrt{x-4}$