

Math 112 Homework for Tuesday, Week 10

1. For each statement, prove or give a concrete counter-example:
 - (a) If $\{a_n\}$ and $\{b_n\}$ both diverge, then $\{a_n + b_n\}$ diverges.
 - (b) If $\{a_n\}$ converges and $\{b_n\}$ diverges, then $\{a_n + b_n\}$ diverges.
 - (c) If $\{a_n\}$ diverges, then so does $\{|a_n|\}$.
2. By reasoning about subsequences, show that $\left(\cos\left(\frac{n\pi}{3}\right)\right)$ diverges.
3. State whether each of the following is True or False. If True, give a justification. If False, give the simplest and most concrete counterexample you can think of. “Monotone” means either monotone increasing or monotone decreasing.
 - (a) If a bounded sequence is monotone, it’s convergent.
 - (b) If a convergent sequence is monotone, it’s bounded.
 - (c) If a convergent sequence is bounded, then it’s monotone.
 - (d) If a sequence is bounded, then it’s convergent.
4. Let π_i be π truncated at the i -th digit. So

$$\pi_1 = 3.1, \quad \pi_2 = 3.14, \quad \pi_3 = 3.141,$$

and so on. Prove, using an ε - N argument that $\lim_{i \rightarrow \infty} \pi_i = \pi$.