

Math 112 Homework for Tuesday, Week 9

1. For each of the following, find the limit or determine that the sequence does not converge. Provide a brief explanation for your answer.

(a) $\lim_{n \rightarrow \infty} \frac{8n^{-2}}{-5n^{-2} + n^{-1}}$

(b) $\lim_{n \rightarrow \infty} \frac{(-1)^n}{3^n}$

(c) $\lim_{n \rightarrow \infty} \sin\left(\frac{n\pi}{4}\right)$.

2. Give an ε - N proof that

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} = 0.$$

3. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence, and define a new sequence $\{b_n\}_{n=0}^{\infty}$ where $b_n = a_{n+1}$. Another way to write this sequence is $\{a_{n+1}\}_{n=0}^{\infty}$. We'll just write $\{a_n\}$ and $\{a_{n+1}\}$ for these sequences.

- (a) Give an ε - N proof that if $\{a_n\}$ converges, then so does $\{a_{n+1}\}$, i.e., if $\lim_{n \rightarrow \infty} a_n$ exists, then so does $\lim_{n \rightarrow \infty} a_{n+1}$.
- (b) Prove (in whatever manner is easiest) that if $\lim_{n \rightarrow \infty} a_n$ converges, then

$$\lim_{n \rightarrow \infty} (a_{n+1} - a_n) = 0.$$

4. Prove that

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0.$$

(Hint: write $n!/n^n$ as the product of n distinct terms, and bound this expression below by 0 and above by a simple function of n . You can then either give a simple ε - N proof or use the squeeze theorem.)