Math 112 Homework for Tuesday, Week 8

1. For the following subsets of $\mathbb{R}$ determine if it is open, closed, or neither. You don't have to write a rigorous proof, but you should give some reasoning for your answer (for example, a sketch).

   (a) $(0, 1) \cup [2, 3]$

   (b) $\{1/n, n \in \mathbb{N}^+\}$

   (c) $[-2, \infty)$

2. For the following subsets of $\mathbb{C}$: (i) sketch each set on a complex plane; (ii) determine if it is open, closed, or neither. Again, you don't have to write a rigorous proof, but you should give some reasoning for your answer.

   (a) $\{z \in \mathbb{C} \colon \mathrm{Im}(z) = 0, 0 < \mathrm{Re}(z) < 1\}$

   (b) $\{z \in \mathbb{C} \colon \mathrm{Im}(z) = 0, 0 \leq \mathrm{Re}(z) \leq 1\}$

   (c) $\{z \in \mathbb{C} \colon -2 \leq \mathrm{Im}(z) \leq 2, 0 \leq \mathrm{Re}(z) \leq 1\}$

3. Show that arbitrary unions of open sets are open. More precisely, let $\{U_\alpha, \alpha \in I\}$ be a family of open subsets of F (F is either $\mathbb{R}$ or $\mathbb{C}$). Prove that $\bigcup_{\alpha \in I} U_\alpha$ is open.