Math 112 Homework for Tuesday, Week 7

1. Compute the following and express your answer in the form $a + bi$ with $a, b \in \mathbb{R}$:

   (a) $\overline{4 - 6i}$.

   (b) $|3 - 4i|$.

   (c) $(3 + 2i)^2$.

   (d) $\mathrm{Im}(2 + 5i + i(3 - 7i) + 17)$.
   (If $z = a + bi$ with $a, b \in \mathbb{R}$, the *imaginary part* of z is $\mathrm{Im}(z) := b$.)

   (e) $(4 + 3i)/(1 - i)$.

   (f) $(6 + 2i)\overline{(6 + 2i)}$.

2. Let F be an ordered field. In class, we proved the triangle inequality:

$$|u + v| \leq |u| + |v|$$

   for all $u, v \in F$. *Follow the method outlined below*, providing solutions to parts (a) and (b), to prove the reverse triangle inequality

$$|x - y| \geq \big|\, |x| - |y| \,\big|$$

   for all $x, y \in F$.

   Method.

   (a) Show $|x - y| \geq |x| - |y|$ by making a substitution $u = \square$, $v = \square$ into the ordinary triangle inequality. The boxes for u and v are to be filled by functions of x and y. (Do not revert to the definition of the absolute value, breaking the problem into cases.) Specify u and v.

   (b) Use part (a) and the fact that $|-a| = |a|$ for all $a \in F$ to show $|x-y| \geq |y|-|x|$.

   From part (b), it follows that $-|x-y| \leq |x|-|y|$. Combined with part (a), we get

$$-|x - y| \leq |x| - |y| \leq |x - y|,$$

   which implies the result: $|x - y| \geq \big|\, |x| - |y| \,\big|$.

3. For $A \subseteq \mathbb{R}$ and $k \in \mathbb{R}$, define

$$kA := \{ka : a \in A\}.$$

Thus, every element of kA has the form ka for some $a \in A$.

Examples:

$$3\{2, 4, 7\} = \{6, 12, 21\} \qquad \text{and} \qquad 4(-1, 2) = (-4, 8).$$

If A is bounded and nonempty, and $k > 0$, prove that

$$\sup(kA) = k \sup(A)$$

as follows:

(a) Show $k \sup(A)$ is an upper bound for kA. (Template for proof: Let $x \in kA$. Clearly explain why this implies $x \leq k \sup(A)$.)

(b) Show that if $t < k \sup(A)$, then t is not an upper bound for kA.