Math 112 Homework for Tuesday, Week 4

1. Let $f\colon A \to B$ be a function. We make two definitions:

   Def. A function $g\colon B \to A$ is a *left inverse for f* if

   $$g(f(a)) = a$$

   for all $a \in A$.

   Def. A function $g\colon B \to A$ is a *right inverse for f* if

   $$f(g(b)) = b$$

   for all $b \in B$.

   (a) Prove that if f has a left inverse $g\colon B \to A$, then f is injective.

   (b) Prove that if f has a right inverse $g\colon B \to A$, then f is surjective.

2. Consider the following two functions

   $$\begin{aligned} f\colon \mathbb{R} \times \mathbb{R} &\to \mathbb{R} \\ (x, y) &\mapsto x \end{aligned}$$

   and

   $$\begin{aligned} g\colon \mathbb{R} &\to \mathbb{R} \times \mathbb{R} \\ x &\mapsto (x, 0). \end{aligned}$$

   (Thus, $f(x, y) = x$ and $g(x) = (x, 0)$.)

   (a) Compute $f \circ g$ and $g \circ f$. Display these functions as we have f and g, above, so that the domain and codomain of each is clear.

   (b) For each of the functions f, g, $f \circ g$, and $g \circ f$, say whether the function is injective, whether it is surjective, and whether it is bijective. (No explanation is required, but you may provide one if you'd like.)

3. (a) Make addition tables for $\mathbb{Z}/n\mathbb{Z}$ for $n = 2, 3, 4$.

(b) Make multiplication tables for $\mathbb{Z}/n\mathbb{Z}$ for $n = 5, 6, 7, 8$.

For convenience, instead of writing an element of $\mathbb{Z}/n\mathbb{Z}$ using brackets, e.g., $[a]$, just write a where $a \in \{0, 1, \dots, n-1\}$. For example, here is the multiplication table for $\mathbb{Z}/4\mathbb{Z}$:

$\bullet$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

.

To save space, let's also leave out the entry for 0. So the multiplication table for $\mathbb{Z}/4\mathbb{Z}$ is

$\bullet$	1	2	3
1	1	2	3
2	2	0	2
3	3	2	1

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