Math 112 Homework for Tuesday, Week 3

1. Let A be the set of all lines in the plane.

   (a) Prove that "is parallel to" is an equivalence relation on A. Describe the equivalence classes.

   (b) Prove that "is perpendicular to" is not an equivalence relation on A.

2. Given a set A, the *power set* of A is the set $\mathcal{P}(A)$ of all subsets of A:

$$\mathcal{P}(A) = \{B \colon B \subseteq A\}.$$

   Is the subset relation $\subseteq$ on $\mathcal{P}(A)$ reflexive? Is it symmetric? Is it transitive? Is it an equivalence relation? In each case, give a proof or a counterexample.

3. Define a relation $\wr$ on $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ by

$$(a, b) \wr (a', b') \quad \Longleftrightarrow \quad a \cdot b' = b \cdot a'.$$

   (a) Prove that $\wr$ is an equivalence relation.

   (b) Describe the equivalence classes of $(0, 1)$, $(1, 1)$ and $(2, 3)$.

   (c) Find a natural identification between the set of equivalences classes and $\mathbb{Q}$.