Math 112 Homework for Friday, Week 3

1. Find a set A and functions $f, g\colon A \to A$ such that $f \circ g \neq g \circ f$.

2. (a) Consider the function $f\colon \mathbb{Z} \to \mathbb{Z}/10\mathbb{Z}$ defined by $f(a) = [a]$. Is f surjective? Is f injective? Explain your reasoning.

   (b) Let $k \in \mathbb{N}$, and define a function $f_k\colon \mathbb{Z} \to \mathbb{Z}/10\mathbb{Z}$ by $f_k(a) = [k \cdot a]$. Notice that the case $k = 1$ is the function f in part (a). For which values of k is the function f_k surjective? (Hint: it may be easier to first determine all values of k for which f_k is *not* surjective.) You should explain your reasoning as best you can, without worrying about a complete proof.

3. Suppose that A and B are sets and $f\colon A \to B$ is a function from A to B. The *image* of a subset S of A under f is the set

$$f(S) = \{f(x)\colon x \in S\}.$$

   Suppose that $C \subseteq A$.

   (a) Prove that $f(A) \setminus f(C) \subseteq f(A \setminus C)$.

   (b) Give a counterexample to show that $f(A) \setminus f(C) = f(A \setminus C)$ is not always true.

   (c) Find a condition on f for which $f(A) \setminus f(C) = f(A \setminus C)$ is always true. Justify your answer with a proof.