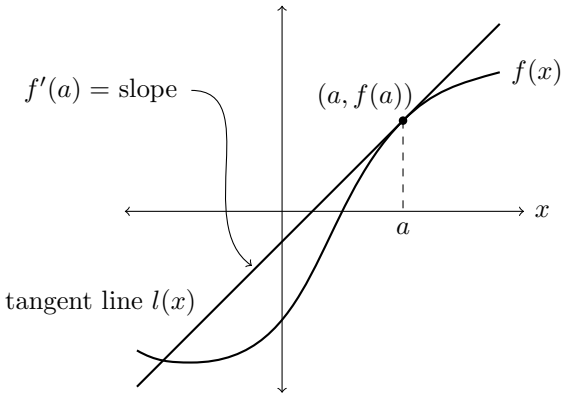
MATH 111, SHEET 4: PRACTICE PROBLEMS

John Lind • September 18, 2017

Let's practice. Our aim here is to think about the meaning of the derivative in answering some questions from the textbook. You should come to class having done the reading and warm-up problems, and ready to tackle the practice problems. *Please bring the textbook to class this week, if you have a copy.*

Here are a few supplementary remarks to think about while you do the reading. You might also examine Sheet 5 for an additional perspective on some of this material.

- The derivative at $x = a$ of a function $f(x)$ is the slope of the tangent line to $f(x)$ at the point $(a, f(a))$:



- If $f(x)$ is a given quantity, then

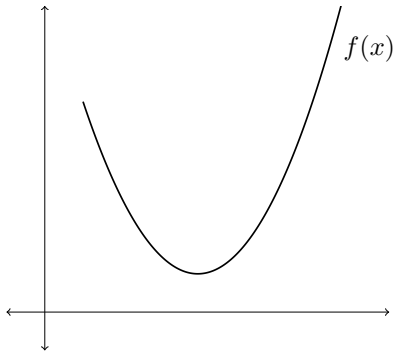
$$\boxed{f'(a) \text{ is the instantaneous rate of change of } f(x) \text{ when } x = a.}$$

  For example, if $f(t)$ is the distance traveled by an object after t units of time, then the derivative $f'(t)$ is the speed of the object at time t.

- Thinking of the process $a \longmapsto f'(a)$ as the association of outputs to inputs, defines the *derivative function* $f'(x)$. We use the variable "x" for its input to show that we are thinking of f' as a function of x.[1]

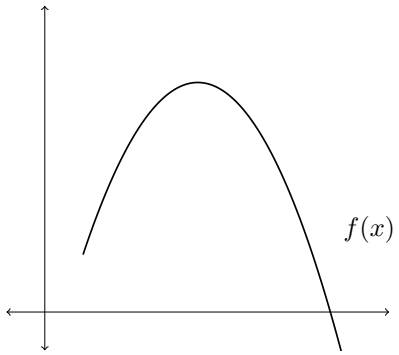
[1]Remember that the derivative function $f'(x)$ is *different from* the function $l(x) = f'(a)(x - a) + f(a)$ whose graph is the tangent line at a particular point $x = a$!

- If the derivative function $f'(x)$ is increasing, then the rate of change of $f(x)$ is increasing. In this situation, we say that $f(x)$ is concave up (think of a bowl filling with fruit or water):



If the slope $f'(x)$ increases as x increases, then f is *concave up*

- If the derivative function $f'(x)$ is decreasing, then the rate of change of $f(x)$ is decreasing. In this situation, we say that $f(x)$ is concave down (think of a hilltop or dome):



If the slope $f'(x)$ decreases as x increases, then f is *concave down*

Reading. §2.1–2.3 of the textbook.

Warm-Up Problems. Do problems §2.2.11 and §2.3.5 from the text, and bring your responses to class on Wednesday.

Practice Problems.

- §2.2: 4, 12, 18, 25, 26, 28
- §2.3: 4, 6, 12, 24, 26, 35, 36, 40, 48

Reading. §2.4 of the textbook.

Warm-Up Problems. Do problems §2.4.3 and 2.4.7 from the text, and bring your responses to class on Friday.

Practice Problems.

- §2.4: 15, 20, 24, 26, 28, 30
- §3.1: 56, 58, 60, 62, 64, 66, 68, 78