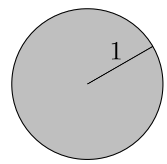
MATH 111, EXPLORATION 4

Due Friday, September 22

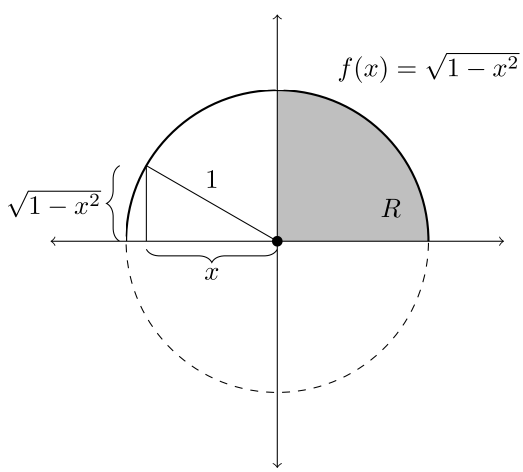
Definition. The number π is the area of a circle of radius one:[1]

$\pi :=$ area of 

You have one task for this week's exploration.

(1) Compute as many digits of π as you can, given your computing power and temporal resources. To do this, you should use the method of over-approximating and under-approximating area that we developed in Explorations 2 and 3. You should also include a brief discussion explaining *how you know that your answer is accurate* (and I don't mean comparing with a value for π that someone else has computed).

Here are a few comments to guide you. By the Pythagorean theorem, the upper perimeter of the circle of radius one centered at the origin is the graph of the function $f(x) = \sqrt{1-x^2}$.



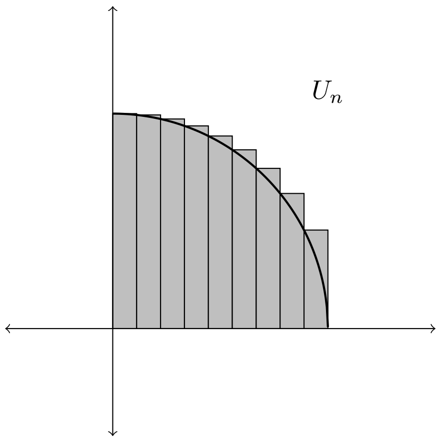
$$x^2 + (\sqrt{1-x^2})^2 = 1^2$$

Let R be the region below the graph of $f(x) = \sqrt{1-x^2}$ where $x \geq 0$ and $y \geq 0$. This is the upper-right quarter of the circle, so

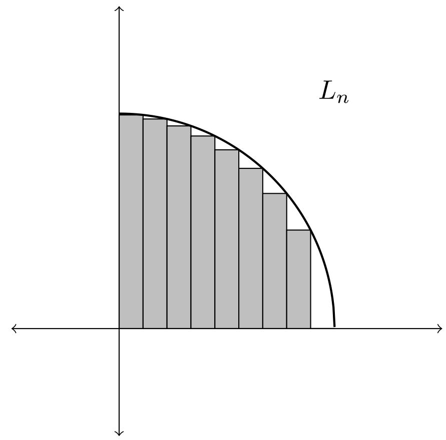
$$\pi = 4 \cdot \text{area}(R).$$

[1] The symbol π is the Greek letter *pi*.

Subdivide the interval $[0, 1]$ into n equal width subintervals, then construct boxes over $\overline{B}_i$ and under $\underline{B}_i$ the graph to find a formula for the upper sum U_n and the lower sum L_n.



The upper sum U_n is the over-approximation of area(R) given by the area of n boxes above the graph.



The lower sum L_n is the under-approximation of area(R) given by the area of n boxes below the graph.

Note: In Mathematica, if you click on "increase precision" under the output of a numerical value, you can specify the number of decimal places that you require. Equivalently, the command

`N[` (an entry) `,d]`

will output the given entry in numerical form to d decimal places.