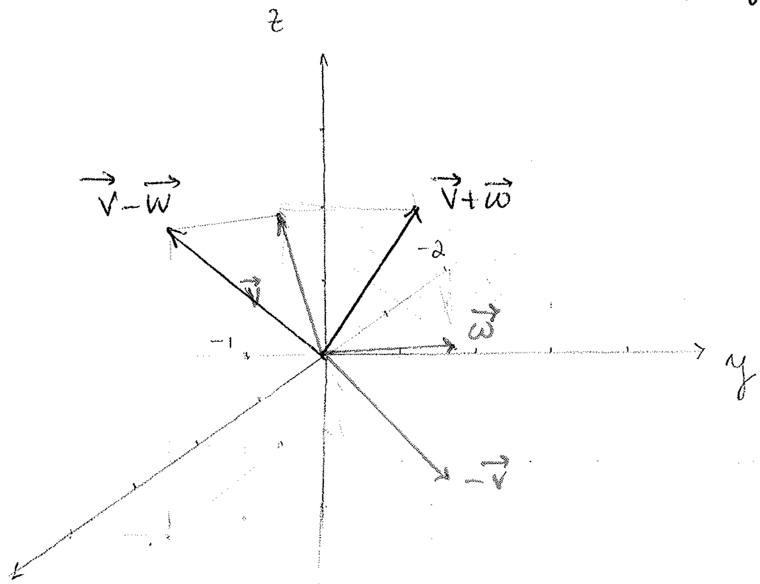
§1.1 (8) Sketch the given vectors $\vec{v}$ & $\vec{w}$, draw $-\vec{v}$, $\vec{v}+\vec{w}$ & $\vec{v}-\vec{w}$

$\vec{v} = (2,1,3)$

$\vec{w} = (-2,0,-1)$.



note $-\vec{v} = (-2,-1,-3)$

$\vec{v}+\vec{w} = (0,1,2)$

$\vec{v}-\vec{w} = (4,1,4)$.

(14) Describe the points in the following configurat:

the plane spanned by $\vec{v_1} = (3,-1,1)$ and $\vec{v_2} = (0,3,4)$.

These are all points/vectors of the form:

$$\alpha v_1 + \beta v_2 = \alpha(3,-1,1) + \beta(0,3,4) =$$

$$= (3\alpha, -\alpha+3\beta, \alpha+4\beta), \quad \text{for } \alpha,\beta \in \mathbb{R}.$$

(18) Describe the points of the line passing through $(-5,0,4)$ and $(6,-3,2)$.

the line is
$$x = -5 + t(6+5) = -5+11t$$
$$y = 0 + t(-3-0) = -3t$$
$$z = 4 + t(2-4) = 2t$$

(28) Do the lines $(x, y, z) = (t+4, 4t+5, t-2)$ and $(x, y, z) = (2s+3, s+1, 2s-3)$ intersect?

If they do then $\exists$ s & t such that:

$$\begin{cases} 2s+3 = t+4 & (1) \\ s+1 = 4t+5 & (2) \\ 2s-3 = t-2 & (3) \end{cases}$$

from (2) $\Rightarrow$ $s = 4t+4$ plug in (1)

$\Rightarrow$ $2s+3 = t+4 \Rightarrow 2(4t+4)+3 = t+4$

$8t+8+3 = t+4$

$8t+11 = t+4$

$7t = -7$

$t = -1$

they intersect at $t = -1$ or $(x, y, z) = (3, 1, -3)$

§ 1.2 Compute $\|\vec{u}\|$, $\|\vec{v}\|$ and $\vec{u}.\vec{v}$

(8) $\vec{u} = 5i - j + 2k$

$\vec{v} = i + j - k$

$\|\vec{u}\| = \sqrt{\vec{u}.\vec{u}} = \sqrt{25+1+4} = \sqrt{30}$

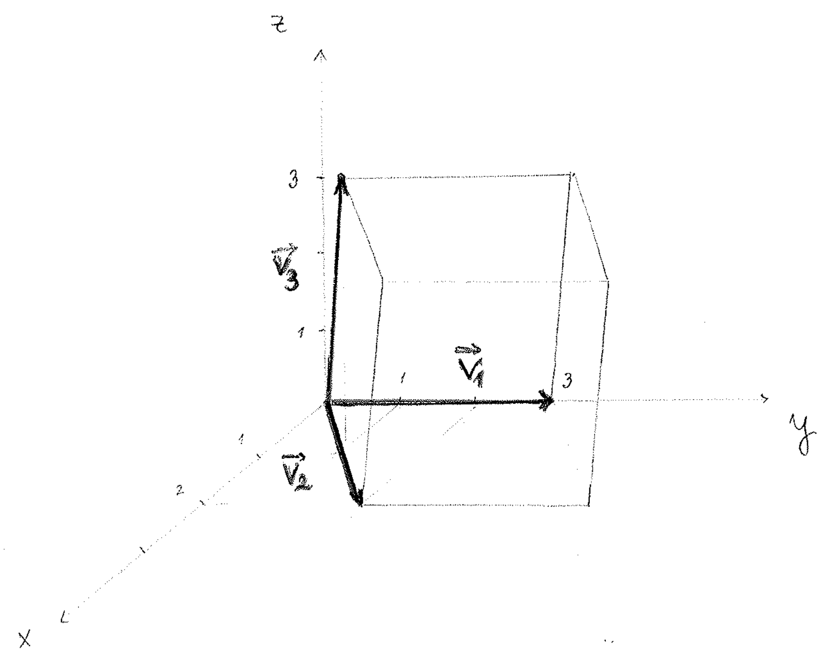
$\|\vec{v}\| = \sqrt{\vec{v}.\vec{v}} = \sqrt{1+1+1} = \sqrt{3}$

$\vec{u}.\vec{v} = 5.1 - 1.1 - 2.1 = 5 - 1 - 2 = 2.$

(14) Let $\vec{v}_1 = (0,3,0)$
$\vec{v}_2 = (2,2,0)$
$\vec{v}_3 = (1,1,3)$.

These three vectors with their tails at the origindetermine a parallelepiped P.

a) Draw P



b) Determine the length of the main diagonal (from the origin to the opposite vertex).

the main diagonal is the sum of the three vectors $\vec{v}_1$, $\vec{v}_2$ & $\vec{v}_3$.

To see this consider first $\vec{v}_1+\vec{v}_2$ (this is the main diagonal of the ~~front~~ bottom face of P) Then $(\vec{v}_1+\vec{v}_2)+\vec{v}_3$ is the main diagonal.

Thus $\vec{v}_1+\vec{v}_2+\vec{v}_3 = (0,3,0)+(2,2,0)+(1,1,3) = (3,6,3)$

the length of $\|\vec{v}_1+\vec{v}_2+\vec{v}_3\| = \sqrt{(3,6,3)\cdot(3,6,3)} = \sqrt{9+36+9} = \sqrt{54}$

(20) Find the projection of $\vec{u} = -i+j+k$ onto $\vec{v} = 2i+j-3k$.

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\underbrace{\|\vec{v}\|^2}_{=\vec{v}\cdot\vec{v}}}\cdot\vec{v} = \frac{(-1,1,1)\cdot(2,1,-3)}{\|(2,1,-3)\|^2}(2,1,-3) =$$

$$= \frac{(-2+1-3)}{(4+1+9)}(2,1,-3) = \frac{-\cancel{4}^{2}}{\cancel{14}_{7}}(2,1,-3) = \left(-\frac{4}{7}, -\frac{2}{7}, \frac{6}{7}\right).$$

(24) Let $\vec{b} = (3,1,1)$ and P be the plane through the origin given by $x+y+2z=0$.

a) Find an orthogonal basis for P (that is $\vec{v}_1, \vec{v}_2 \in P$ such that $\vec{v}_1 \perp \vec{v}_2$ & $\vec{v}_1 \neq 0$, $\vec{v}_2 \neq 0$).

the vectors in P are of the form (x,y,z) with $x = -y-2z$, thus of the form $(-y-2z, y, z)$.
Thus two vectors in P can be $(-1,1,0)$ and $(-2,0,1)$.
These however are not orthogonal as $(-1,1,0)\cdot(-2,0,1) \neq 0$.
instead consider $\vec{v}_1 = (-1,1,0)$ & $\vec{v}_2 = (-2,0,1) - \text{proj}_{\vec{v}_1}(-2,0,1)$
now $\vec{v}_1$ & $\vec{v}_2$ are orthogonal.

Note: there are a lot of ways to solve this problem. all you need to find is two vectors that are in P & have dot product 0. Another way to solve this is:

If you set $\vec{v}_1 = (-1, 1, 0)$ & $\vec{v}_2 = (a, b, c) \Rightarrow$ we have:

$$\left.\begin{array}{l} \vec{v}_1 \cdot \vec{v}_2 = 0 \\ \vec{v}_2 \in P \end{array}\right\} \Rightarrow \begin{array}{l} -1a + b = 0 \\ a + b + 2c = 0 \end{array}$$

use these two equations to find some a, b, c.

$$\Rightarrow \left| \begin{array}{l} a = -b \\ a + b + 2c = 0 \end{array} \right. \Rightarrow \left| \begin{array}{l} a = b \\ 2(a+c) = 0 \end{array} \right. \Rightarrow \left| \begin{array}{l} a = b \\ a = -c \end{array} \right.$$

$\Rightarrow$ pick $a = 1 \Rightarrow b = 1, \; c = -1$.

$\vec{v}_2 = (1, 1, -1)$

(check that $\vec{v}_1 \perp \vec{v}_2$:))

thus $\vec{v}_1, \vec{v}_2$ form an orthogonal basis for P. where $\vec{v}_1 = (-1, 1, 0)$, $\vec{v}_2 = (1, 1, -1)$.

b) find the orthog. proj of $\vec{b}$ onto P, that is; find

$\text{proj}_{\vec{v}_1} \vec{b} + \text{proj}_{v_2} \vec{b}$.

$$\text{proj}_{\vec{v}_1} \vec{b} = \frac{\vec{b} \cdot \vec{V}_1}{\|\vec{V}_1\|^2} \cdot \vec{V}_1 = \frac{\vec{b} \cdot \vec{V}_1}{\vec{V}_1 \cdot \vec{V}_1} \vec{V}_1 = \frac{(3,1,1)(-1,1,0)}{(-1,1,0)(-1,1,0)} \vec{V}_1 =$$

$$= \frac{-3+1+0}{1+1+0}(-1, 1, 0) = \frac{-\cancel{2}^{\,1}}{\cancel{2}_{\,1}}(-1, 1, 0) = (1, -1, 0)$$

$$\text{proj}_{\vec{v_2}} \vec{b} = \frac{\vec{v_2}\cdot\vec{b}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2} = \frac{(1,1,-1)(3,1,1)}{(1,1,-1)(1,1,-1)}(1,1,-1) =$$

$$= \frac{3+1-1}{1+1+1}(1,1,-1) = (1,1,-1)$$

$$\text{proj}_P \vec{b} = (1,-1,0) + (1,1,-1) = (2,0,-1).$$

(26) Find the line through $(3,1,-2)$ that intersects and is perpendicular to the line:

$$\left.\begin{aligned} x &= -1+t \\ y &= -2+t \\ z &= -1+t. \end{aligned}\right\} \quad (-1,-2,-1) + t\underbrace{(1,1,1)}_{\vec{v}}.$$

the line we are looking for will be given by:

$$\left.\begin{aligned} x &= 3+s\omega_1 \\ y &= 1+s\omega_2 \\ z &= -2+s\omega_3 \end{aligned}\right\} \quad (3,1,-2) + s\underbrace{(\omega_1,\omega_2,\omega_3)}_{\vec{\omega}}.$$

as the two lines are perp. to each other $\Rightarrow \vec{v} \perp \vec{w}$ ie. $\vec{v}\cdot\vec{w} = 0$.

an as they intersect we have:

$$\begin{array}{ll|l}
(1) & \omega_1 + \omega_2 + \omega_3 = 0 & (\text{as } \vec{v}\cdot\vec{w} = 0) \\
(2) & 3 + s\omega_1 = -1+t & \\
(3) & 1 + s\omega_2 = -2+t & \\
(4) & -2 + s\omega_3 = -1+t &
\end{array}$$

sum up (2), (3), (4) to get.

$$3+1-2+\underbrace{s(w_1+w_2+w_3)}_{0 \text{ by } (1)} = -1-2-1+3t$$

$$2+s\cdot 0 = -4+3t$$

$$3t=6$$

$$\underline{t=2}$$

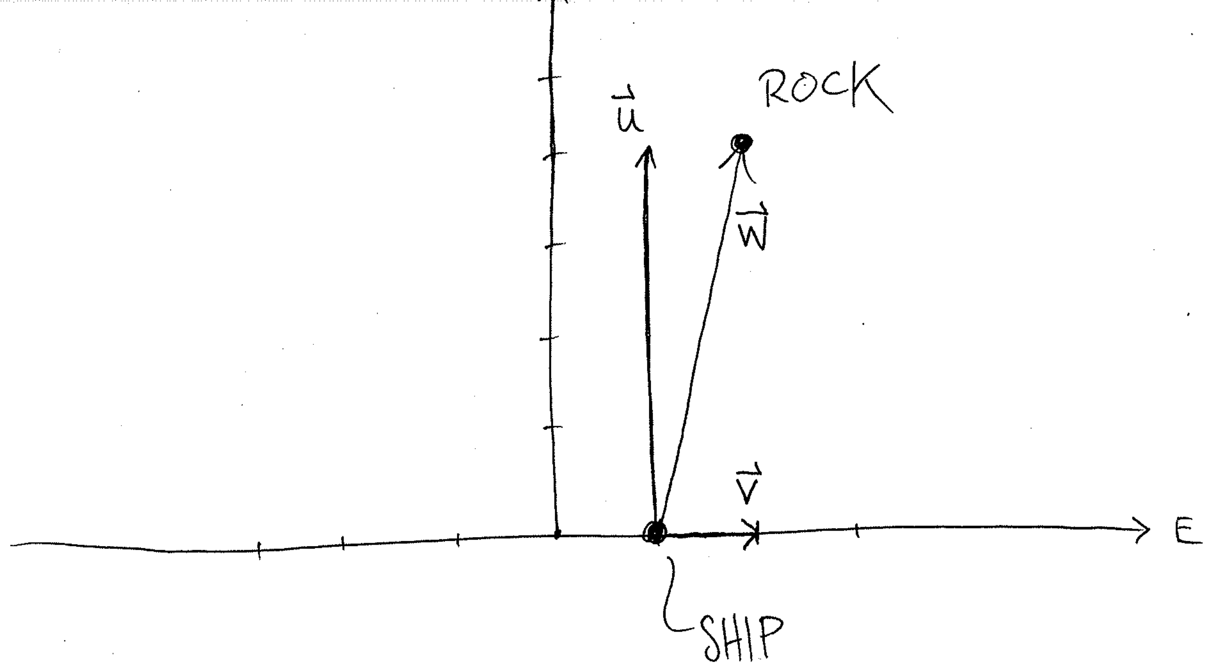
thus the lines intersect at
$$x=-1+2=1$$
$$y=-2+2=0$$
$$z=-1+2=1.$$

thus we have two poinst from the line that we are looking for: $(1,0,1)$ and $(3,1,-2)$.

Thus the line is:
$$x=1-s(3-1)=1-2s$$
$$y=0-s(1-0)=-s$$
$$z=1-s(-2-1)=1+3s.$$

(30) Suppose there is a ship at position $(1,0)$ on a nautical chart (with north in the positive y direct.) sights a rock at $(2,4)$.
The ship is pointing due north and travelling at speed 4 n.miles/h.
There is a current flowing due east at 1 n.m/h.

a) If there is no current what vector $\vec{u}$ would represent the velocity of the ship?



THE VELOCITY OF THE SHIP (WITH THE WATER HOLDING STILL) IS $\vec{u} = (0, 4)$

(B) THE VELOCITY OF THE CURRENT IS

$$\vec{v} = (1, 0)$$

(C) THE TOTAL VELOCITY OF THE SHIP IS

$$\vec{w} = \vec{u} + \vec{v} = (0, 4) + (1, 0) = (1, 4)$$

(D) AFTER ONE HOUR THE SHIP WOULD BE AT:

$$(x, y) = \text{ORIGINAL LOCATION} + \text{DISPLACEMENT}$$
$$= (1, 0) + \Delta t \cdot (\text{VELOCITY}) = (1, 0) + 1 \cdot (1, 4) = (2, 4)$$

(E) THE CAPTAIN SHOULD CHANGE COURSE BECAUSE THE BOAT WILL HIT THE ROCK AT TIME $t = 1$.

(F) IF THE ROCK WERE AN ICEBERG, IT WOULD BE MOVED BY THE CURRENT TO $(3, 4)$ AT $t = 1$, SO THE SHIP WOULD MISS IT.

§1.3

(2) Evaluate the determinants:

a) $$\begin{vmatrix} 2 & -1 & 0 \\ 4 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = 2.3 + (4-6) = $$
$$= 6 - 2 = \underline{\underline{4}}.$$

b) $$\begin{vmatrix} 36 & 18 & 17 \\ 45 & 24 & 20 \\ 3 & 5 & -2 \end{vmatrix} \underset{\text{Switch row 1 & 2}}{=} -\begin{vmatrix} 45 & 24 & 20 \\ 36 & 18 & 17 \\ 3 & 5 & -2 \end{vmatrix} \underset{\text{row 2 - row 1}}{=} -\begin{vmatrix} 45 & 24 & 20 \\ -9 & -6 & -3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= (+3)\begin{vmatrix} 45 & 24 & 20 \\ 3 & 2 & 1 \\ 3 & 5 & -2 \end{vmatrix} \underset{\text{row 3 - row 2}}{=} (+3)\begin{vmatrix} 45 & 24 & 20 \\ 3 & 2 & 1 \\ 0 & 3 & -3 \end{vmatrix} =$$

$$= (+9)\begin{vmatrix} 45 & 24 & 20 \\ 3 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix} \underset{\text{row 1 - 15x row 2}}{=} (+9)\begin{vmatrix} 0 & -6 & 5 \\ 3 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix} \underset{\text{switch row 1 and row 2}}{=}$$

$$= -9\begin{vmatrix} 3 & 2 & 1 \\ 0 & -6 & 5 \\ 0 & 1 & -1 \end{vmatrix} = -9.3\begin{vmatrix} -6 & 5 \\ 1 & -1 \end{vmatrix} = -27(6-5) = \underline{\underline{-27}}$$

c) $$\begin{vmatrix}1&4&9\\4&9&16\\9&16&25\end{vmatrix} \underset{\substack{\text{row 3 –}\\ \text{2×row 2}}}{=} \begin{vmatrix}1&4&9\\4&9&16\\1&-2&-7\end{vmatrix} \underset{\substack{\text{row 3}\\ +\\ \text{row 1}}}{=} \begin{vmatrix}1&4&9\\4&9&16\\2&2&2\end{vmatrix} = (2)\begin{vmatrix}1&4&9\\4&9&16\\1&1&1\end{vmatrix}$$

$$\underset{\substack{\text{row 1 – row 3}\\ \text{row 2 – 4row 3}}}{=} 2\begin{vmatrix}0&3&8\\0&5&12\\1&1&1\end{vmatrix} = 2\begin{vmatrix}3&8\\5&12\end{vmatrix} = 2(36-40) = -2.4 = \underline{-8}$$

d). $$\begin{vmatrix}2&3&5\\7&11&13\\17&19&23\end{vmatrix} \underset{\substack{\text{row 2 – (3) row 1}\\ \text{row 3 – row 2}}}{=} \begin{vmatrix}2&3&5\\1&2&-2\\10&8&10\end{vmatrix} = 2\begin{vmatrix}2&3&5\\1&2&-2\\5&4&5\end{vmatrix} \underset{\substack{\text{row 3}\\ -\\ \text{row 1}}}{=}$$

$$= 2\begin{vmatrix}2&3&5\\1&2&-2\\3&1&0\end{vmatrix} = 3.2\begin{vmatrix}3&5\\2&-2\end{vmatrix} - 2(+1)\begin{vmatrix}2&5\\1&-2\end{vmatrix}$$

$$= 6(-6-10) - 2(-4-5) =$$

$$= +6(-16) + 2(9) = -6.16 - 18 =$$

$$= -6.16 - 6.3 = -6(16-3) = -6.13$$

$$= \underline{-78}$$

(12) Describe all unit vectors orthog. to the vectors

$\vec{u} = 2i - 4j + 3k$

$\vec{v} = -4i + 8j - 6k$.

Note that $\vec{v} = (-2)\vec{u}$, thus $\vec{u}$ & $\vec{v}$ lie on the same line.

The vectors that are orthogonal to both $\vec{u}$ & $\vec{v}$ line on the plane(s) for which $\vec{u}$ (or $\vec{v}$) is a normal vector.

The equat. of the plane P for which $\vec{u}$ is normal & P passes through the origin is:

$2x - 4y + 3z = 0.$

Thus all vectors of length 1 lying on P and all planes $\parallel$ to P (or alternatively in the span of $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$) are orth. to both $\vec{u}$ & $\vec{v}$

(22) Find the intersection of the two planes P_1 & P_2 with eq.:

$3(x-1) + 2y + (z+1) = 0$ &

$(x-1) + 4y - (z+1) = 0.$

The line of intersection lies both on P_1 & P_2, and thus it is orthogonal to both $\vec{n}_1$ & $\vec{n}_2$, where $\vec{n}_i$ is the normal vector for P_i $(i = 1, 2)$.

Thus if the line is: $\vec{a} + t\vec{v}$

$\vec{v} = \vec{n_1} \times \vec{n_2}$.

$\vec{n_1} = (3,2,1)$
$\vec{n_2} = (1,4,-1)$

$$\Rightarrow \vec{v} = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 4 & -1 \end{vmatrix} = i\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} =$$

$$= i(-6) - j(-4) + k10$$

$$= -6i + 4j + 10k$$

We need a point on P_1 & P_2 that is $\vec{a} = (x_0, y_0, z_0)$ should satisfy both:

$$3(x_0 - 1) + 2y_0 + (z_0+1) = 0$$
$$(x_0 - 1) + 4y_0 - (z_0+1) = 0.$$

such a point is $\vec{a} = (1, 0, -1)$.

so the line would be $(1, 0, -1) + t(-6, 4, 10)$.

(28) Find an equation for the plane P that passes through the point $(2,-1,3)$ and is perpendic. to the line: $\vec{v} = (1,-2,2) + t(3,-2,4)$.

The plane is $A(x-2) + B(y+1) + C(z-3) = 0$.

where (A,B,C) is the normal, but we know $\vec{v} \parallel (A,B,C)$, as $\vec{v} \perp P \Rightarrow (A,B,C) = (3,-2,4)$

$\Rightarrow P$ is $3(x-2) + (-2)(y+1) + 4(z-3)$.

(34) Find the distance from the point $(2,1,-1)$ to the plane $x-2y+2z+5=0$.

from the formula on p. 43:

$$\text{distance} = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$$

where $(x_1, y_1, z_1) = (2,1,-1)$

and the plane is $Ax+By+Cz+D=0$.

$$\Rightarrow \text{distance} = \frac{|1.2+(-2).1+2.(-1)+5|}{\sqrt{1^2+(-2)^2+2^2}} = \frac{3}{\sqrt{9}} = 1$$

(38) Given vectors $\vec{a}$ and $\vec{b}$, do the equations $\vec{x} \times \vec{a} = \vec{b}$ and $x.\vec{a} = \|\vec{a}\|$ determine a unique vector $\vec{x}$?

First note that for this problem to make sense we need $\vec{a} \perp \vec{b}$, because:

$$\vec{a}.\vec{b} = \vec{a}.(\vec{x}\times\vec{a}) = 0.$$

case 1) $\vec{a}=0$

$$\left.\begin{array}{l} \vec{x}\times\vec{a} = \vec{x}.\vec{0} = \vec{0} = \vec{b} \\ \Rightarrow \vec{b}=0. \\ \vec{x}.\vec{a} = \vec{x}.\vec{0} = 0 = \|0\| \end{array}\right\} \text{these hold } \forall x!$$

thus there is no unique x in this case.

case 2.1) $\vec{a} \neq 0$, $\vec{b} = 0$

this means $\vec{x} \times \vec{a} = \vec{0}$ ie. $\vec{x} \parallel \vec{a}$, $x = \lambda a$, $\lambda \in \mathbb{R} \setminus \{0\}$.

then $\vec{x} \cdot \vec{a} = \|a\|$ implies that $\vec{x} = \vec{a} / \|\vec{a}\|$ as $a.a = \|a\|$

thus we have a unique solution.

case 2.2) $\vec{a} \neq 0$, $\vec{b} \neq 0$.

we can always find a basis in which $\vec{a} = (n, 0, 0)$ for some $n \in \mathbb{R} \setminus \{0\}$.

$\|\vec{a}\| = n$. (wlog we can pick $\vec{a} = (n, 0, 0)$.

$\vec{x} = (m_0, m_1, m_2)$ for some $m_1, m_2, m_3 \in \mathbb{R}$.

then $\vec{a} \cdot \vec{x} = n . m_0 = \|a\| = n \Rightarrow \underline{m_0 = 1}$

we know that $\vec{x} \times \vec{a} = \vec{b} = (b_0, b_1, b_2)$

$$\vec{x} \times \vec{a} = \begin{vmatrix} i & j & k \\ 1 & m_1 & m_2 \\ n & 0 & 0 \end{vmatrix} = n \begin{vmatrix} j & k \\ m_1 & m_2 \end{vmatrix} = n(jm_2 - km_1)$$

$$= (0, m_2 . n, -m_1 . n) = (b_0, b_1, b_2)$$

$$\Rightarrow m_1 = {}^{-b_2}/n$$

$$m_2 = b_1/n$$

$$\Rightarrow \vec{x} = (1, -b_2/n, b_1/n).$$

Geometrically: note that ~~as~~ if $\vec{a} = 0$ we get to informat. about $\vec{x}$ so $\vec{x}$ can be any vector.

if $\vec{b} = 0$, $\vec{a} \neq 0$

then $\vec{x}$ & $\vec{a}$ lie on the same line & are mutiples of each other.

Also note that $\text{proj}_{\vec{a}} \vec{x} = \frac{\vec{a}}{\|a\|}$ but as $\vec{x} \in \text{span}\{\vec{a}\}$

$$\Rightarrow \text{proj}_{\vec{a}} \vec{x} = \vec{x} = \vec{a}/\|a\|.$$

if $\vec{a} \neq 0$, $\vec{b} \neq 0$

a, b & $a \times b$ form a basis for $\mathbb{R}^3$ (orthogonal basis as $a \perp b$ & $\{a, b\} \perp a \times b$)

then $\text{proj}_{\vec{a}} \vec{x}$, $\text{proj}_{\vec{b}} \vec{x}$, $\text{proj}_{\overrightarrow{a \times b}} \vec{x}$ = completely determine the vector $\vec{x}$ that is,

$$\vec{x} = \text{proj}_{\vec{a}} \vec{x} + \text{proj}_{\vec{b}} \vec{x} + \text{proj}_{a \times b} \vec{x}.$$

§1.4| Find the spherical coordinates of the point
(2) $(x,y,z) = (\sqrt{6}, -\sqrt{2}, -2\sqrt{2})$.

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{6+2+8} = 4$$

$$\varphi = \cos^{-1}(z/\rho) = \cos^{-1}\left(\frac{-2\sqrt{2}}{4}\right) = \frac{3\pi}{4}.$$

recall that

$$x = \cos\theta \sin\varphi\, \rho \Rightarrow \sqrt{6} = \cos\theta\, 2\sqrt{2}$$

$$y = \sin\theta \sin\varphi\, \rho \Rightarrow -\sqrt{2} = \sin\theta\, 2\sqrt{2}$$

$$\Rightarrow \cos\theta = \sqrt{3}/2$$
$$\sin\theta = -1/2$$

$$\Rightarrow \theta = 330^\circ = \pi \cdot \frac{11}{6}.$$

(10) Describe the following solids using inequalities. State the coordinate system used.

a) A cylindrical shell 8 units long with inside diam. 2 units and outside 3 units.

(cylindrical coordinates).

$$0 \leq z \leq 8$$
$$2 \leq \sqrt{x^2+y^2} \leq 3.$$

b) A spherical shell with inside radius 4 units & outside radius 6 units:

(spherical coordinates).

$4 \leq \rho \leq 6$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi$.

c) A hemisphere of diam. 5.

(spherical coordinates).

$0 \leq \rho \leq 5$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi/2$.

d) A cube of side 2.

(cartesian coord.).

$0 \leq x \leq 2$

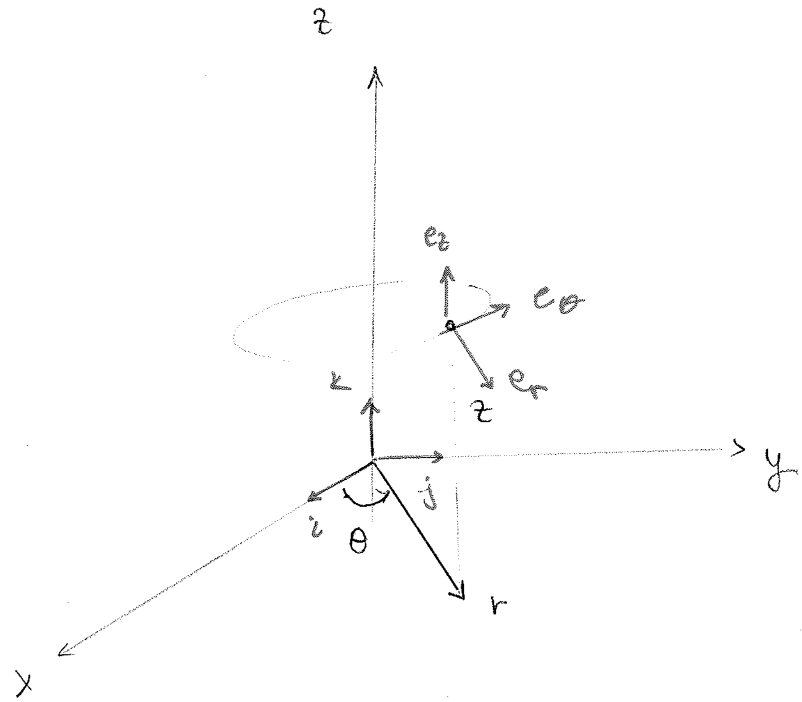
$0 \leq y \leq 2$

$0 \leq z \leq 2$.

(12) Using cylindrical coord. and the orthonormal vectors e_r, e_θ and e_z

a) express e_r, e_θ & e_z in terms of i, j, k & (x, y, z).

b) calculate $e_\theta \times j$ both analytically (using part a)) and geometrically.



$e_z = k$

$e_r = i\cos\theta + j\sin\theta$ (we know this from calc 2).
b/c the polar coord. of $\forall$ point in x-y plane are $(r\cos\theta, r\sin\theta)$
and we want $\|e_r\| = 1 \Rightarrow r = 1$.

e_θ is the unit vect. tangent to the the curve parametrized by θ, with r & z fixed.

to find e_θ use the fact that e_θ, e_r & e_z form an orthonormal basis, that is:

$e_\theta \perp e_r$ & $e_\theta \perp e_z$

$$\Rightarrow e_\theta = e_r \times e_z = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} i & j \\ \cos\theta & \sin\theta \end{vmatrix}$$

$$= i\sin\theta - j\cos\theta$$

$= (\sin\theta, \cos\theta, 0)$ note that $\|e_\theta\| = 1$.

thus: $e_r = \cos\theta\, i + \sin\theta\, j = (\cos\theta, \sin\theta, 0)$

$e_z = k = (0, 0, 1)$

$e_\theta = \sin\theta\, i - \cos\theta\, j = (\sin\theta, -\cos\theta, 0)$.

b). $e_\theta \times j = \begin{vmatrix} i & j & k \\ \sin\theta & -\cos\theta & 0 \\ 0 & 1 & 0 \end{vmatrix} = (-1) \begin{vmatrix} i & k \\ \sin\theta & 0 \end{vmatrix} = (-\sin\theta)k \in \operatorname{span} \vec{k}$

geometrically e_θ does not depend on z. That is e_θ is the same whether $z = 0$ or $z > 0$.

$\Rightarrow$ if $z = 0 \Rightarrow e_\theta \in$ x-y plane, but so is j

$\Rightarrow e_\theta \times j$ is the vector perpendicular to the plane spanned by e_θ & j ie the x-y plane, thus $e_\theta \times j$ is parallel to k. (which is altso perpendicular to the x-y plane).