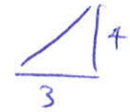
8.1

8)

Use green theorem,

work = $\iint_P \left( \frac{dQ}{dx} - \frac{dP}{dy} \right) dx\,dy =$ T : 


$\int_T (3x + 4y^2)\,dx + (10xy)\,dy = \underline{16}$

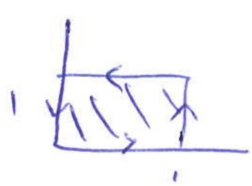
20)

because for green's theorem we need $\frac{dP}{dy}$ and $\frac{dQ}{dx}$

to be $C^1$. for example

$\frac{dP}{dy} = \frac{-2y^2 - x^2 - y^2}{\sqrt{x^2+y^2}} \notin C'$

21)




$\int_0^1\int_0^1 4x^3 - 2y\,dx\,dy = \int_0^1 x^4 - 2yx \Big|_0^1 dy$

$= \boxed{0}$

27)

$r(0) = r(\frac{\pi}{2}) \rightarrow$ one loop is from $\theta = 0$ to $\theta = \frac{\pi}{2}$

$$\text{area} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \underbrace{(3\sin(2\theta))^2\, d\theta}_{r^2 d\theta = xdy - ydx}$$

$$\rightarrow = \frac{9}{2}\int_0^{\frac{\pi}{2}} \frac{1+\cos(4\theta)}{2}\, d\theta = \boxed{\frac{9\pi}{8}}$$

8.2)

6)

$$\text{Curl } \vec{F} = (2y)\hat{i} + 2z\hat{j} + \hat{k}$$

$$\nabla \times F = (2y, 2z, 1)$$

Use parametrization

$$\Phi(u,v) = (\cos u \sin v, \sin u \sin v, \cos v)$$

$$T_u = (-\sin u \sin v, \cos u \sin v, 0)$$

$$T_v = (\cos u \cos v, \cos v \sin u, -\sin v)$$

Surface integral: $\iint_S \nabla \times F \cdot ds =$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi} (2\sin u \cos u \sin^3 v + 2\sin u \sin^2 v \cos v + \sin v \cos v)\, du\, dv$$

$$= \int_0^{\frac{\pi}{2}} 2\pi \sin v \cos v\, dv = \boxed{\pi}$$

Now compare this with line integral

$c'(t) = (-\sin t, \cos t, 0)$

$$\int_{\partial s} \vec{F} \cdot d\vec{s} = \int_0^{2\pi} \cos^2 t \, dt = \boxed{\pi}$$

verified!

13)

boundry of this surface is $\begin{cases} z = 0 \\ x^2 + y^2 = 1 \end{cases}$

Using polar coordinates

$c(\theta) = (\cos\theta, \sin\theta, 0)$ $\qquad 0 \leq \theta \leq 2\pi$

$c'(\theta) = (-\sin\theta, \cos\theta, 0)$

$$\cancel{\int_{\partial s} \vec{F} \cdot d\vec{s} = \int_0^{2\pi} (}$$

$$\int F \cdot d\vec{s} = \int_0^{2\pi} (-\sin\theta, \cos\theta, 0) \cdot (\cos\theta, \sin\theta, 0) \, d\theta$$

$= \underline{0}$

18)

Using stokes' theorem

boundry $\begin{cases} 2z^2+y^2=10 \\ x=0 \end{cases}$

$\vec{F}=(0, e^x, -yz)$

using polar coordinates

$x=0 \quad y=\cos\theta \quad z=2\sin\theta \quad 0\leqslant\theta\leqslant 2\pi$

$$\iint_S (\nabla\times F)\,ds = \int_{\partial S} \vec{F}\cdot d\vec{s} = \int_0^{2\pi} F(c(t))c'(t)\,dt$$

$$= \int_0^{2\pi} (0, 1, -\cos\theta\times 2\sin\theta)\cdot(0, -\sin\theta, 2\cos\theta)\,d\theta$$

$= 0$

or! it has no bondry $\rightarrow$ integral $=0$

8.3) 2)

a) $F=\nabla(x\cos(xy)+c)$

b) does not exist

c) $F=\nabla(x^2\cos y + x\cos y + c)$

10)

a) $\vec{r}$ is radial $\rightarrow$

$$\frac{d}{dx}\left(\frac{1}{r}\right) = \frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}} =$$

$$\rightarrow \nabla\left(\frac{1}{r}\right) = \frac{-(x, y, z)}{\|r\|^3}$$

b) Since $\vec{F}$ is gradient then the work is path independent

$$\ell(\infty) - \ell(r_0) = \lim_{r\to\infty} \frac{1}{r} - \frac{1}{\|r_0\|} = \frac{1}{\|r_0\|}$$

18)

a) $F = \pi(x^2 + xy^2 + y\cos x + c)$

c) $F = \pi(xy^3 + x + y + c)$

8. 4)

4)

left side

$$\int_0^1\int_0^{2\pi}\int_0^{\pi} \rho^2 \sin\phi \, d\phi \, d\theta \, dr = \frac{4\pi}{3}$$

Righ side:

$$\iint_{\partial W} (-y, x, z) \cdot (x, y, z) \, dS =$$

$$\int_0^{\pi}\int_0^{2\pi} \cos^2\phi \sin\phi \, d\theta \, d\phi = \frac{4\pi}{3}$$

10) C)

$$\operatorname{div} F = \frac{dF_1}{dx} + \frac{dF_2}{dy} + \frac{dF_3}{dz} = 1+1+1 = 3$$

$$\iint_{\partial W} F \cdot dS = 3\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^1\int_{r^2}^1 r \, dz \, dr \, d\theta = \frac{3\pi}{4}$$

14)

$\operatorname{div} F = 8y+1$

$$\iint_S F \cdot dS = \iiint_W \operatorname{div} F \, dV = \iiint_W 8y+1 \, dV$$

$$= \int_{-3}^{3} \int_{y^2}^{9} \int_0^{x} 8y+1 \, dz \, dx \, dy = \frac{972}{5}$$

16)

$$\partial S = \begin{matrix} x^2+y^2 \le 1 \\ 0 \le z \le 1 \end{matrix}$$

use divergeic theorem

$$\iint_{\partial S} F \cdot dS = \iiint (x^2+y^2)^2 dV = \int_0^{2\pi} \int_0^1 \int_0^1 r^5 \, dr \, dz \, d\theta$$

$$= \frac{\pi}{3}$$