

MATH 112 REVIEW FOR FINAL EXAM

There will be an in-class, closed book/notes/internet final exam 6–9 pm on Wednesday, May 9, in Vollum Lecture Hall. The following is a precise description of what you are expected to know for the exam.

1. What is a *relation* on a set? An *equivalence relation*?
2. There will probably be an *induction* proof of some sort on the exam.
3. Know the meaning of the following terms used when talking about functions: *domain*, *codomain*, *image (range)*, *surjective*, *injective*, *bijective*, *composition*, *inverse*.
4. What is a *field*?
5. What is  $\mathbb{Z}/n\mathbb{Z}$ ? Is it a field?
6. What are the *order* axioms?
7. What is the *completeness axiom* for the real numbers?
8. Be able to define the following terms associated with sets of real numbers: *upper bound*, *lower bound*, *sup*, *inf*, *max*, and *min*.
9. What are the *complex numbers*? Be able to add, multiply, and divide complex numbers, expressing your result in the form  $a + bi$ .
10. What is the *polar form* of a complex number? What is the *argument* and *modulus*? Be able to convert between two forms of a complex number. Know what happens to the argument and modulus when you multiply two complex numbers.
11. What is an *open ball* in  $\mathbb{R}$  or  $\mathbb{C}$ ? What is an *open set* in  $\mathbb{R}$  or  $\mathbb{C}$ ?
12. What is a *closed set* in  $\mathbb{R}$  or  $\mathbb{C}$ ?
13. For a real or complex sequence  $\{a_n\}$ , what does it mean to say  $\lim_{n \rightarrow \infty} a_n = a$ ?
14. Be able to give an  $\varepsilon$ - $N$  proof for the limit of a simple sequence.
15. What is a *monotone sequence*?
16. Be able to prove that *every convergent sequence is bounded*.
17. State and prove the *monotone convergence theorem*.
18. Be able to prove that if  $\{a_n\}$  and  $\{b_n\}$  are convergent sequences, then so is  $\{a_n + b_n\}$ , and  $\lim(a_n + b_n) = \lim a_n + \lim b_n$ . (This is an  $\varepsilon/2$  proof.)
19. What does it mean for a series to be *absolutely convergent*? *Conditionally convergent*?

20. Know the *tests for convergence* of series and how to use them. A summary of the tests and the solutions to the practice problems on sequences and series appears here: [series tests](#).
21. Know how to sum a *geometric series* (being careful where the series starts).
22. For a real or complex function  $f$ , what does it mean to say  $\lim_{z \rightarrow a} f(z) = L$ ?
23. Be able to give an  $\varepsilon$ - $\delta$  proof for the limit of a simple function.
24. What does it mean to say a function is *continuous* at a point?
25. What is the definition of the *derivative* of a function at a point?
26. Know the two big theorems about powers series: the first having to do with the existence of a *radius of convergence*, and the second having to do with differentiation.
27. Be able to use the ratio test to find the radius of convergence of a power series.
28. Know the Taylor series for the exponential (denoted  $\exp(z)$  or  $e^z$ ), the sine, and cosine functions. From these, be able to show for all  $z \in \mathbb{C}$  that
  - $\exp'(z) = \exp(z)$ ,  $\cos'(z) = -\sin(z)$ , and  $\sin'(z) = \cos(z)$ ;
  - $\exp(iz) = \cos(z) + i \sin(z)$ .
29. Be able to compute the *Taylor polynomial* of a given order or the *Taylor series* for a function.